ON SCALING IN HEAD INJURY RESEARCH

by

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Introduction

Data regarding impact tolerance levels in head injury research have to be based on experiments performed on cadavers or animals for obvious reasons. From the differences in shape and size between animal brains and the human brain follows that no direct conclusions regarding tolerance levels for man can be made from animal tests. The shape problem is generally handled by choosing a test animal (model) whose brain shape resembles that of a human (prototype) as closely as possible. In most cases primates have been chosen. The size problem has been handled by the use of some scaling technique, the most natural of which was proposed by Ommaya et al., (1967). The assumptions made in their paper are cited below:

1. The brain acts as an elastic medium.
2. Brain tissue is homogeneous and isotropic in nature.
3. The density of this tissue is equal in model and prototype.
4. Model and prototype brains are geometrically similar, through one scale factor.
5. Injury is the result of shear strains exceeding a certain value.
6. The skull is very stiff, such that deformations of the skull do not contribute heavily to the strains in the enclosed brain.
7. Stiffness factors of the contained brains in model and prototype are equal.

The third and seventh of these assumptions might be generalized to the statement that material properties in the model and the prototype are equal. Anisotropic effects, if any, could also be incorporated in such a statement. This is a very natural assumption and it is largely supported by experimental work, e.g. the comprehensive study at West Virginia University, (1970). Furthermore, the assumption of geometrical similarity could easily be extended to cover also inhomogeneities, so that the second assumption would in fact cause no restrictions on the choice of possible models in itself. In the same manner one could of course allow significant skull deformation if geometric similarity and material equality were extended to hold true about skull bone as well.

When discussing scaling techniques it is not really necessary to detail the exact injury causing mechanism, be it intracranial strains, stresses, forces or pressures which are belived to exceed their respective critical values. The critical value is a property of the material as long as rate effects can be neglected. Usually rate effects on the strength of materials are fairly unimportant in intervals where the rate can change by at most a factor of ten, cf. Löwenhielm (1974). Here, therefore, it is assumed that the appropriate critical value is simply and solely a property of the material and as such then already assumed equal in model and prototype. The fifth assumption therefore does not impose any restrictions on the choice of model. The first assumption, stating elastic behaviour of the involved materials (thereby

excluding for instance time dependent behaviour), can however not be supported by experimental data. The viscoelastic nature of brain matter is clearly shown by e.g. Schuck and Advani, (1972) and Ljung, (1975). Let us exemplify the problems which arise if viscosity is incorporated as a significant material parameter in the study. Say that we wish to compare geometrically similar systems exposed to actions which in some sense are similar. Then the parameters describing the geometry (the positions of white matter, grey matter, skull bone, etc) can be expressed by means of one single quantity, R, of dimension length, typical (and different) for each different system, and a set of dimensionless quantities that are identically the same for all systems regarded. This means that all distances of a system scale linearly with R and that they need not appear in the continued reasoning.
Similarly, the action of the forces that provoke motion of the systems should be expressible by means of one single quantity, say an acceleration a(t). Since equality of material behaviour was assumed, only one characteristic set of material constants is needed. Let us denote the characteristic propagation velocity of equivoluminal waves by c_T, the characteristic ratio between the propagation velocities of equivoluminal and irrotational waves by k and the characteristic kinematic viscosity by ν. Then, assuming that we wish to study the shear strain in the brain at a given point as a function of time, t, we may surmise that the expression

$$\gamma = \gamma\left[t;\ R;\ c_T,\ k,\ \nu;\ a(t)\right]$$

can be used for all systems regarded. Since c_T, k and ν are identical for all systems, they are shown explicitly only for the special purpose to investigate the relative significance of elasticity and viscosity.

According to the Buckingham Pi Theorem (cf. Baker et al., 1973) the shear strain γ can be written as a function of a complete set of independent dimensionless groups, formed from the variables and parameters on which γ depends. Thus we can write

$$\gamma = f\left(\frac{c_T t}{R}\ ;\ \frac{\nu}{c_T R}\ ;\ \frac{Ra}{c_T^{\,2}}\right)$$

or

$$\gamma = g\left(\frac{\nu t}{R^2}\ ;\ \frac{\nu}{c_T R}\ ;\ \frac{R^3 a}{\nu^2}\right)$$

It is immediately clear, from the appearance of the middle argument $\nu/c_T R$ in the two expressions for the shear strain, that a consistent scaling law, expressible by means of one scaling factor only, cannot be found. However, at the two asymptotic ends, zero viscosity and zero rigidity, we obtain

$$\gamma = f\left(\frac{c_T t}{R}\ ;\ 0\ ;\ \frac{Ra}{c_T^{\,2}}\right)$$

and

$$\gamma = g\left(\frac{\nu t}{R^2}\ ;\ \infty\ ;\ \frac{R^3 a}{\nu^2}\right)$$

respectively. Thus in both cases a simple scaling law exists. In the first case time, t, scales linearly with R, provided that Ra/c_T^2 is a fixed function of $c_T t/R$ for all systems regarded. In the second case time scales linearly with R^2, provided that $R^3 a/\nu^2$ is a fixed function of $\nu t/R^2$. It should be noted that the requirements that Ra/c_T^2 should be a fixed function of $c_T t/R$ in the first case and that $R^3 a/\nu^2$ should be a fixed function of $\nu t/R^2$ in the second case are themselves implications of the scaling laws for time and distances.

It is now natural to assume that the first scaling law mentioned is approximately applicable when ν/Rc_T is small and the second law when ν/Rc_T is large. The meaning of "small" and "large" for a certain required accuracy has to be extracted from the character of the function γ. Since γ is a functional of the provoking function a(t) it is natural that the result of such an investigation will yield different results for different types of functions a(t). Furermore, since for each given class of similar systems ν and c_T are fixed, it is also possible to relate the result to the magnitude of R.

It is immediately obvious how the scaling law should be modified if a typical velocity v(t) had been chosen instead of the acceleration a(t) to describe the outer action on the system. Then, in the first case v/c_T should be a fixed function of $c_T t/R$ for all systems regarded and, in the second case, Rv/ν should be a fixed function of $\nu t/R^2$.

In the present investigation three different kinds of actions on an idealized skull-brain model will be discussed. They give rise to:
1. Transient motion following a short duration impact,
2. transient motion following a long duration impact and
3. periodic motion in steady state.

The discussion will be based on numerical calculations for a simple model of the skull-brain system developed by Ljung, (1975). Calculations will be made for three different skull sizes, approximating those of man, chimpanzee and squirrel monkey.

Calculations

The model used consists of a rigid, infinitely long cylindrical shell of inner radius R. The shell is filled with a viscoelastic material of density λ, which is described by a first order Kelvin-Voigt model. The two parameters of this model are the kinematic viscosity, ν, and the shear modulus, G. If the shell is exposed to a sudden change in angular velocity, so that it starts to rotate with a constant velocity v_0 around its axis the tangential displacement component, u_1, of the viscoelastic material at radius r will be (cf. Ljung, 1975)

$$u_1(\rho,t) = v_0 \left[\rho t + 2 \sum_{i=1}^{\infty} \frac{J_1(\rho\gamma_i)}{\gamma_i J_0(\gamma_i)} \cdot \exp\left(-\frac{\gamma_i^2}{R^2} t\right) \cdot \frac{\sin \sqrt{\frac{\gamma_i^2}{R^2} c_T^2 - \frac{\gamma_i^4}{R^4} \frac{\nu^2}{4}}\, t}{\sqrt{\frac{\gamma_i^2}{R^2} c_T^2 - \frac{\gamma_i^4}{R^4} \frac{\nu^2}{4}}} \right] \quad (1)$$

where $c_T = \sqrt{G/\lambda}$ is the propagation velocity for equivoluminal waves, $\rho = r/R$ is the relative radius, $J_\mu(z)$ denotes a Bessel function of the first kind and γ_i are the zeros of $J_1(z)$. The displacement relative to the rotating shell is

$$u = u_1 - v_0\,\rho t = u_0 \tag{2}$$

and the shear in the viscoelastic material at the inner surface of the shell is

$$\left(\frac{\partial u}{\partial r}\right)_{r=R} = \left(\frac{\partial u_0}{\partial r}\right)_{r=R} \tag{3}$$

The situation where the cylinder starts to rotate with a constant angular velocity is representative of what happens after a short duration impact i.e. where the impact duration is much shorter than the natural period of the system. Fig. 1 shows in dimensionless form the shear at the inner surface of the shell as a function of time for some different shell radii.

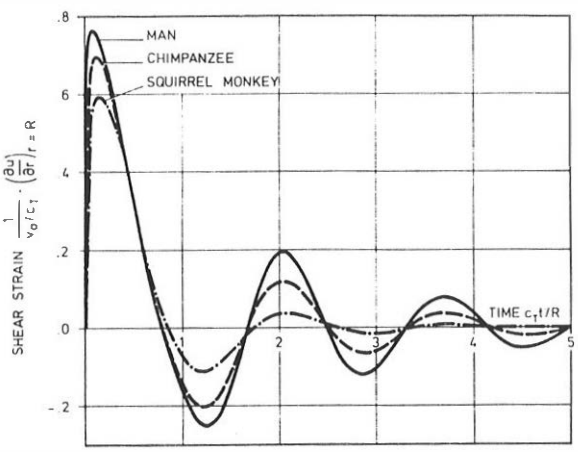

Fig. 1. Mathematical simulations of the response to a short duration impact. The shear at the inner surface of the shell is shown as a function of time for man, chimpanzee and squirrel monkey. The axes are scaled according to the elastodynamic scaling law.

Throughout the calculations three values of the shell radius have been used: R = 0.10 m, R = 0.07 m and R = 0.04 m. These values are typical for the skull size of man, a big ape (e.g. a chimpanzee) and a smaller monkey (e.g. a squirrel monkey) respectively. The values $\nu = 0.009\ m^2/sec$ and $c_T = 1.3$ m/sec are taken from a previous experimental investigation (Ljung, 1975).

Let us now turn our attention to impact durations which are long in comparison to the natural period of the system. In this case one has to look for the response to a suddenly applied constant rotational acceleration of the shell. If this acceleration at radius r = R is a_0 we obtain the shear at r = R from the relation

$$\left(\frac{\partial u}{\partial r}\right)_{r=R} = \frac{a_0}{v_0}\int_0^t \left(\frac{\partial u_0}{\partial r}\right)_{r=R} dt \quad . \tag{4}$$

This shear is shown in dimensionless form in Figure 2 as a function of time for some different shell radii.

Finally, we wish to study the steady state response to a periodic motion of the skull obeying the relation

$$u(1,t) = |A| \cdot \sin \omega t. \tag{5}$$

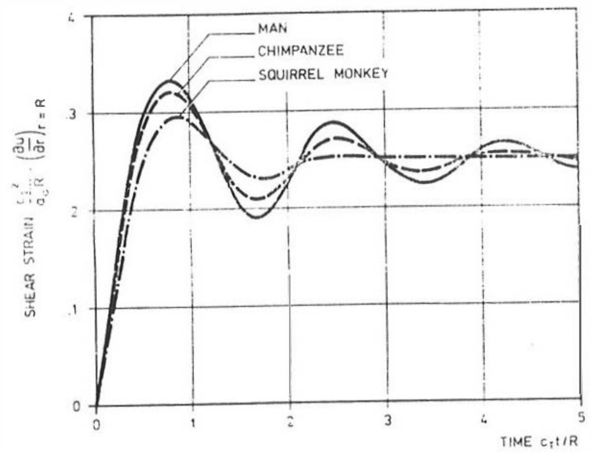

Fig. 2. Mathematical simulations of the response to an idealized long duration impact. The shear at the inner surface of the skull is shown as a function of time for man, chimpanzee and squirrel monkey. The axes are scaled according to the elastodynamic scaling law.

If the result is to be expressed by the shear at the inner surface of the shell

$$\left(\frac{\partial u}{\partial r}\right)_{r=R} = |A| \cdot |B(\omega)| \cdot \sin(\omega t+\phi(\omega)), \tag{6}$$

the simplest way to obtain the resulting amplitude $|B(\omega)|$ and phase $\phi\ (\omega)$ is via the impulsive transfer function of the system, i.e. the Laplace transform of the system response to a unit impulse. This is found to be

$$H(s) = 2 \cdot \sum_{i=1}^{\infty} \frac{}{s^2 + \frac{\gamma_i^2}{R^2} \nu s + \frac{\gamma_i^2}{R^2} c_T^2} . \tag{7}$$

Now

$$|B(\omega)| = |H(j\omega)| \tag{8}$$

and

$$\phi(\omega) = \arg H(j\omega) = \arctan \frac{\operatorname{Im} H(j\omega)}{\operatorname{Re} H(j\omega)} , \tag{9}$$

where $j = \sqrt{-1}$. These two quantities are shown in Fig. 3 as a function of dimensionless frequancy for some different values of the shell radius R. For the benefit of those who wish to interpret the diagram as a Bode plot. logarithmic axes have been used.

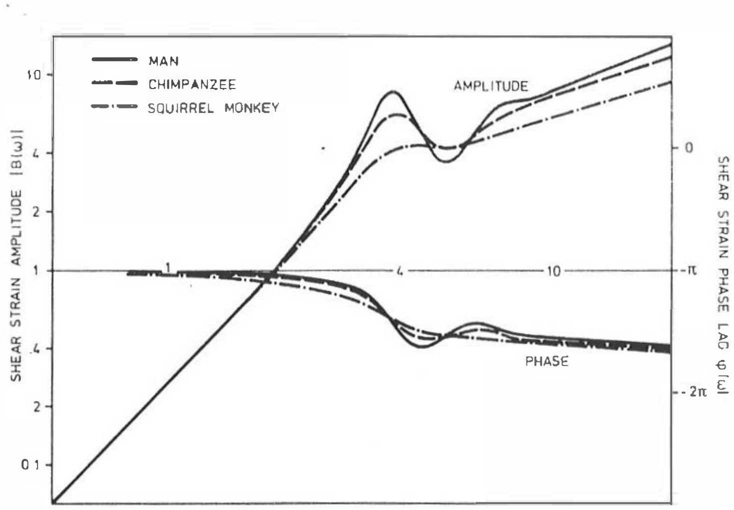

Fig 3. Mathematical simulations of the response to sinusoidal excitation in steady state. The shear at the inner surface of the skull is shown as a function of grequency for man, chimpanzee and squirrel monkey. The axes are scaled according to the elastodynamic scaling law.

Discussion

In all cases studied the dimensionless quantity $\nu/Rc_T < 0.18$, i.e. it is small compared to $2/\gamma_1 \approx 0.52$. Consequently, the dimensionless forms used in Figs. 1-3 pertain to a system which is essentially elastic in nature. These dimensionless forms also agree with the scaling procedure proposed by Ommaya et al., (1967), It is however easily seen from the figures that the legitimacy of this procedure is not entirely obvious, since different values of the shell radius R yield different results. The extent to which the studied viscoelastic cases approximate the pure elastic case ($\nu/Rc_T=0$) and the pure viscous case ($\nu/Rc_T=\infty$) can be found from Figs. 4 and 5. The role of the radius R (the absolute size of the studied system) is clearly demonstrated. Obviously. the bigger the system we study, the less pronounced will the influence of the viscosity be on the system performance. Conversely, a very small viscoelastic system will behave almost entirely as a purely viscous one. The cases studied are however truly viscoelastic in the sense that they do not closely resemble any one of the two limiting cases. Thus the solution in the case of the squirrel monkey does not resemble the viscodynamic solution at all and the amplitude differences between the solution in the case of man and the elastodynamic case are pronounced. In the latter cases, however, one might argue that the essence of the solutions is the same.

If we first turn our attention towards the responses to periodic excitation depicted in Fig. 3, we can immediately conclude that scaling attempts regarding the amplitudes in such experiments may well lead to quite unsatisfactory results. At the fundamental resonance peak the difference in amplitude is more that 25 % when the skull radius is changed from that of man to that of chimpanzee. When the same comparison is made between man and squirrel monkey the difference is more than 75 %. Even though the agreement is quite good between the positions of the resonance peaks, it will obviously be dangerous to draw any quantitative conclusions from model experiments of this kind. As a consequence, inferences regarding tolerance levels will be equally hard to get. On the other hand, it would clearly be possible to determine the location of the main resonances from model experiments.

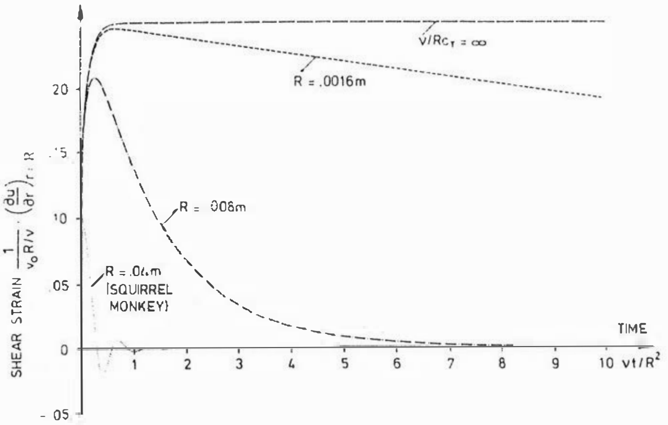

Fig.4. The influence of the system size as expressed by the radius R on the degree of viscous behaviour. The axes are scaled according to the viscodynamic scaling law. The system is subjected to a short duration impact.

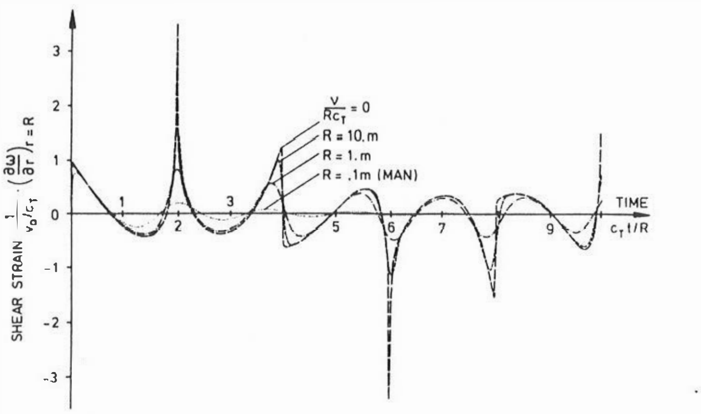

Fig. 5. The influence of the system size on the degree of elastic behaviour. The axes are scaled according to the elastodynamic scaling law. The system is subjected to a short duration impact.

In Figs. 1 and 2 the responses to transient excitation are shown. It is immediately clear from the figures that the scaling law chosen leads to good agreement as far as scaled times are involved. This is of course in direct agreement with the aforementioned fact that resonance frequencies determined in model experiments are good estimates of the true ones. The assumed elastodynamic scale laws will also yield good estimates (within 5 %) of the amplitudes, if the shell radius R is restricted to the range 0.07 - 0.10 m. In the case of long duration impacts one might even extend this range on the basis of the following lines of reasoning. The maximum amplitude is reached during the first overshoot of the response. The duration of this overshoot is (in this slightly modified case) solely determined by the lowest natural frequency of the system, and is consequently much shorter than the load pulse duration. Now, most injury mechanism models take into account the fact that biological tissue can withstand stresses of much higher amplitude if the duration of the pulse is short than if it is long, see e.g. Löwenhielm, (1974). When discussing long duration impacts (Fig. 2) it would therefore be defensible to look more at the finally settled amplitude level than at the maximum amplitude during the overshoot period due to the fact that the overshoot is fairly modest. It is true that the settling time of the response increases as the shell radius decreases, but at the same time the overshoot is decreased. It would then be reasonably safe to assume that scaling could be used at long duration impacts even from small monkeys to man, at least when tolerance levels are sought.

At short duration impact, as suggested by a study of Fig. 1, the maximum amplitude, on the other hand, should be decisive, since the response is dominated by the first overshoot. If scaling is attempted from squirrel monkey to man under these circumstances one could expect the resulting tolerance levels to be approximately 20 % too high. Such a possibility has been mentioned in a dissertation by Löwenhielm (1977). It would be possible, although somewhat dubious, to extend the feasible scaling range by the following kind of reasoning. It was shown previously that in the elastodynamic case time scales linearly with R if the provoking function (in this case the acceleration) fulfils certain requirements (Ra/c_T^2 should be a fixed function of $c_T t/R$). In the viscodynamic case time scales linearly with R^2 provided that $R^3 a/\nu^2$ is a fixed function of $\nu t/R^2$. This means that the amplitude of the provoking acceleration in the first case scales linearly with $1/R$ and in the second case with $1/R^3$. In the viscoelastic case at hand one could then imagine a scaling law of the following form: Time scales linearly with R^{2-n}, where n is a number in the range 0-1, provided that $(Ra/c_T^2)^n \cdot (R^3 a/\nu^2)^{1-n}$ is a fixed function of $(c_T t/R)^n \cdot (\nu t/R^2)^{1-n}$. In the elastodynamic case n = 1, while in the viscodynamic case n = 0. One should be aware of the fact that this scaling law changes the scaling of the time parameter from the so far used elastodynamic one, which was shown to yield good approximations. In the case of the presently discussed skull-brain model one should choose $n \approx 0.8$ to obtain acceptable scaling results in the range $0.04 \text{ m} < R < 0.10 \text{ m}$.

Concluding remarks

The appropriateness of the simple scaling laws for elastodynamic systems has been investigated for two different sizes of the model skull. It was found that the applicability of these laws is dependent not only on the ratio between the prototype and model skull size (expressed by R), but also on the absolute sizes. Thus the same scaling law would be far less successful in the range, say, $R < 0{,}01$ m than in the range of present interest, $R < 0{,}04$ m even if the ratio between prototype and model skull size had been the same. For very small radii, say $R < 0{,}01$ m, the scaling laws for viscodynamic systems would be the suitable ones.

Some uncertainty still prevails regarding the legitimacy of using the scaling laws for elastodynamic systems in the range considered since the value of the critical parameter ν/Rc_T depends not only on R but also on the material parameters ν and c_T. At experimental determination of ν (Ljung, 1975) it was found that a rather large scatter was prevalent. This may, however, reflect not only the fact that variations in biological matter are normally substantial, but also that the system response is rather insensitive to variations of ν. This apparent insensitivity was also demonstrated in the paper referred to, and implies indirectly that the scaling laws for elastodynamic systems should be appropriate.

Differences in shape also play a role, a problem which however has to be considered in a less general manner than the scope of this paper allows for. The significance of the skull shape is closely connected to the kind of injury produced, and should be considered separately in the light of the proposed injury mechanism.

To sum up, it has been shown that the simple scaling laws valid for elastodynamic problems should be expected to give a fair accuracy at attempts to translate results from chimpanzee to man, and, if a certain degree of caution is exercized, even at scaling from squirrel monkey to man. The scaling laws are more adaptable to experiments which rather directly simulate head impacts in practice, than to indirect experiments such as periodic excitations of brain motion.

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