A Deformation Energy Approach to Normalizing PMHS Response Data and Developing Biofidelity Targets for Dummy Design

Bruce R. Donnelly, Kevin M. Moorhouse, Heather H. Rhule, Jason A. Stammen

Abstract Generating a target for the quantitative assessment of crash dummy biofidelity based on normalized post-mortem human subject (PMHS) response data is important for dummy design and development. Normalization is the process of modifying a set of PMHS response data to represent the response of a typical sized human, for example the 50th percentile male. A biofidelity target is the normalized response against which the response of a dummy is quantitatively compared. A mechanistic and statistically based viscoelastic methodology is presented for normalizing PMHS response data based on the applied force, the thoracic deflection and the deformation energy at maximum deflection. A force versus deflection biofidelity target is generated consisting of a mean response curve with a two-dimensional standard deviation ellipse tolerance. This paper provides a detailed explanation of the method, application to an example data set and comparisons to previous methods of normalization using the ellipse coefficient of variation as a measure of similitude.

Keywords PMHS, normalization, scaling, biofidelity, dummy

I. INTRODUCTION

The process of normalizing post-mortem human subject (PMHS) response data from impact testing to obtain a representation of the typical, or average, human response has been an important part of anthropometric crash dummy design and development for many years. Normalization is the process of mathematically modifying the response data from a set of PMHS subjects to a standard human size. Generally the PMHS response data is normalized to the 50th percentile male.

This normalized response is used as a standard against which the response of an anthropometric dummy is compared to assess the dummy biofidelity. A quantified measure of biofidelity is desirable to provide the basis for an objective decision as to the ability of a dummy to assess vehicle crash protection for a human of similar size.

In 1984 Eppinger, et al. [1] presented a methodology for normalizing PMHS data based on the ratio of the whole body mass of a subject to the “standard” total body mass (50th percentile male). The methodology is based on dimensional analysis in which equal density and equal modulus of elasticity are assumed and mass is scaled. Additional scale factors for length (deflection, displacement) force, acceleration and time are derived from the original scale factors. This mass based methodology assumes that all subject response will be related directly to the whole body mass. Clearly force, deflection and kinematic responses from widely varying sizes and shapes of humans are not likely to be related solely by whole body mass. Mertz, et al. [2] presented an impulse-momentum and stiffness-based normalization method in 1984. The effective subject mass was calculated utilizing the applied force versus time history, impulse, and the velocity of the thorax of the subject. Stiffness was derived from a ratio of characteristic lengths (e.g., chest breadth or depth for a thoracic impact) by assuming constant modulus and geometric similarity. Scale factors were developed from ratios of the solution for the single degree-of-freedom (DOF) differential equation for a linear elastic system which represented a large impacting mass such as a sled type of impact. Viano [3] improved upon the Mertz methodology in 1989 by expanding the derivation of scale factors to the two DOF linear elastic system which better represents pendulum-type impacts where the striking and struck masses are more nearly equal. Again the scale factors were developed from ratios of the solution to the system of differential equations.

Moorhouse [4] presented a modification to the Mertz/Viano normalization methodology by using the integral of the force versus deflection curve, the deformation energy, to develop an elastic effective stiffness directly from the force versus deflection response data rather than from characteristic subject dimensions.

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Moorhouse also compared the effectiveness of the various normalization methods in force versus deflection space using a modified coefficient of variation measure for standard deviation ellipses taking one half of the area of the ellipse divided by the product of force and deflection at each data point. This quantitative comparison indicated that effective stiffness collapsed the force versus deflection curves toward each other more effectively than the other methods.

The generation of a characteristic response from PMHS data has been addressed by many authors. The use of straight line segments to create a corridor that envelops the PMHS data has often been utilized to establish a target for dummy response [5-7]. Another methodology is to reduce the set of PMHS data curves to a mean response curve which itself becomes the dummy target [8-9]. In addition to a mean response curve Maltese et al. [10] calculated plus and minus standard deviation at each point in the time history curves to generate a statistically-based corridor. Lessley, et al., [11] presented a method for normalizing force and deflection separately and creating a force versus deflection corridor of plus and minus one standard deviation around a mean curve based on the maximums of the PMHS forces and deflections. Nusholtz et al., [12-13] have focused on a more rigorous statistical approach for creating the PMHS mean response time histories and the comparison of a corresponding dummy response utilizing the auto and cross correlations of the time series data to generate phase, amplitude and shape factors quantifying the similarities and/or differences. These latter investigators also made it clear that if a PMHS data set does not meet the essential requirements of impulse-momentum, the data should not be used in the creation of a typical response target.

The use of the standard deviation ellipse to define a statistically-based and quantifiable biofidelity target was first presented by Shaw et al. [14] to describe the thoracic response of a set of PMHS in low speed pendulum type impacts. The cumulative area within the set of standard deviation ellipses defines the plus and minus one standard deviation variation for the two-dimensional force versus deflection response. If the cumulative difference in response of a dummy from the normalized target response, under the same test conditions, is within the area of the standard deviation ellipses, the dummy response is within plus and minus one standard deviation. If the cumulative difference in dummy response is within the area of two standard deviation ellipses, it is within two standard deviations, and so forth.

This study builds upon the work of Mertz, Viano and Moorhouse as well as Nusholtz to develop normalizing scale factors based on a two DOF viscoelastic model of the PMHS thorax in which the response data meet the requirements of impulse-momentum. This study uses PMHS data from thoracic pendulum type tests at 4.5 m/s performed by Rhule, et al., [15] to illustrate the methodology. The deformation energy of each PMHS is averaged to obtain the best estimate for the typical adult subject. A time scale factor, λ_t, is developed from the damped natural frequency obtained from a least squares fit of the force versus deflection data for a two element Kelvin/Voigt viscoelastic model. The effective mass and velocity at maximum thoracic deflection are found for each subject from the deformation energy, conservation of energy and conservation of momentum, and then the scale factors, λ_m and λ_V, are calculated. The scale factor for force, λ_F, is found from averaged impulse and momentum and the scale factor for deflection, λ_D, is found from the averaged deformation energy. A mean force versus deflection curve is generated along with a standard deviation ellipse target. The results are compared to results from the non-normalized data, the Eppinger method, the Mertz/Viano method and the Moorhouse method using the modified ellipse coefficient of variation approach of Moorhouse.

This deformation energy normalization methodology has also been applied to a novel PMHS data set by Stammen, et al [16] to develop a mean adult torso bending response that is then scaled to a large child size in order to develop a biofidelity design target for a large child dummy.

II. METHODS

The term deformation energy is defined in this study to be the work done in the deflection of the subject thorax from the time of initial deflection to the time of maximum deflection. This includes the energy that is absorbed elastically and will be returned during the unloading portion of the event and the energy that is dissipated viscously as heat. Energy is also transferred from the impactor to the subject as kinetic energy which results in a velocity change of the subject. Energy losses at the boundary due to friction are neglected. The energy and momentum of the system are conserved if the deformation energy, as determined from the applied force and the resulting deflection of the thorax, are included in the energy balance.

The two main assumptions of this deformation energy normalization methodology are (1) PMHS mechanical responses to impact are normally distributed and the central mean theory holds making the means and standard deviations of the PMHS responses the best estimates of a typical human response and (2) the use of averaged PMHS response data from a set of identical impact tests can be used to linearly normalize the individual response data from each subject toward the mean response.

The procedure for deformation energy normalization of the PMHS response data from a set of identical tests is:

1. Calculate the deformation energy by integrating the force versus deflection curve of each PMHS up to the time of maximum thoracic deflection. For the example data set the applied force as measured by the impactor load cell and the thoracic deflection as measured by a forty gage chestband were used in this integration. Note that the force and deflection time histories must have the same sampling rate and synchronized time zero in order to parametrically calculate the energy integral.

$$E_{Dj} = \sum_{i=1}^{M} F_{ji} * \Delta D_{ji}$$

   E_{Dj} = deformation energy of each subject j
   i = the time increment number starting with the first deflection increment after time zero
   M = the increment at maximum thoracic deflection
   F_{ji} = applied force at increment i
   ΔD_{ji} = thoracic deflection increment

2. Assuming that the impactor velocity and the subject velocity are the same at the time of maximum thoracic deflection, conservation of energy, including the deformation energy from above, and conservation of momentum are used to find the effective mass and the common velocity at the time of maximum thoracic deflection for each subject (two equations and two unknowns).

$$\frac{1}{2}M_pV_0^2 = \frac{1}{2}M_pV_p'^2 + \frac{1}{2}M_{sj}V_{sj}'^2 + E_{Dj}$$

$$M_pV_0 = M_pV_p' + M_{sj}V_{sj}'$$

   M_p =mass of impactor
   V_0 =velocity of impact
   V_p =velocity of impactor at maximum deflection
   M_{sj}=effective mass of the subject j thorax (unknown)
   j = subject number
   E_{Dj} = deformation energy of the subject j
   V_{sj} =velocity of the subject j thorax at time of maximum deflection and assumed equal to V'_P (unknown)

3. Calculate the impulse for each subject up to the time of maximum thoracic deflection by integrating the force time histories.

$$I_j = \sum_{i=1}^{M} F_{ji} * \Delta t_{ji}$$

   I_j = impulse
   Δt_{ji}= time increment from i-1 to i

4. Check on the validity of the data for each subject by comparing the subject impulse to the subject momentum at the time of maximum thoracic deflection using the effective mass and the common velocity from step 2 above. If the difference between the impulse and momentum exceeds 20% [12] the data set from the subject should not be included.
5. Calculate a subject specific scale factor (normalizing factor) for deformation energy by averaging the set of maximum deformation energies for each subject and dividing the average maximum deformation energy by the individual maximum deformation energy of the subject.

$$\lambda_E = \frac{\frac{1}{N}\sum_{j=1}^{N} E_{Dj}}{E_{Dj}}$$

   λ_E=scale factor for deformation energy
   N= number of subjects in the sample

6. The typical effective mass is found by averaging the ratios of effective mass to the whole body mass for each subject and multiplying this average ratio times the 50th percentile whole body mass to get the typical effective mass. This scale factor for effective mass is found by dividing the typical effective mass by the individual effective mass of the subject.

$$\overline{M_S} = \sum_{j=1}^{N} \frac{M_{Sj}}{M_{SBj}} * M_{SB50}$$

$$\lambda_M = \frac{\overline{M_S}}{M_{Sj}}$$

$\overline{M_S}$= typical effective mass
M_{SB50}= whole body mass of the 50th percentile male
λ_M= scale factor for effective mass

7. Fit a second order polynomial that passes through zero to the deflection time history for each PMHS, up to the time of maximum deflection.
8. Calculate a least squares fit of the equation for a Kelvin-Voigt two-element viscoelastic solid to the force versus deflection data for each subject using the polynomial from step 7 above for deflection and rate of deflection (see Appendix A. for the derivation). This will result in an estimate of the stiffness constant, *k*, and the viscous constant, *c*, for each subject as well as the scale factors λ_k and λ_c.
9. Find the damped natural frequency for each subject from the solution to the system of differential equations for the viscoelastic two DOF system.

$$f_j = \sqrt{M_{rj}k_j - \frac{1}{4}M_{rj}^2 c_j^2}$$

$f_j =$ damped natural frequency of each subject
$M_{rj} =$ the mass ratio $(M_p + M_{Sj})/(M_p \cdot M_{Sj})$ for each subject
$k_j =$ the stiffness constant for each subject
$c_j\ =$ the viscous constant for each subject

10. Calculate a subject specific scale factor for time by averaging the damped natural frequencies for each subject and dividing each subject frequency by the average frequency of the subjects (note that the time scale factor is the reciprocal of the frequency scale factor).

$$\bar{f} = \frac{1}{N}\sum_{j=1}^{N} f_j$$

$$\lambda_{Tj} = \frac{f_j}{\bar{f}}$$

$\bar{f}$=averaged damped natural frequency
λ_{Tj}=scale factor for time for each subject

11. The average momentum change is calculated using the typical effective mass and the average common velocity from step 6 and step 2.

$$\overline{Mom_S} = \overline{M_S} \cdot \overline{V_S'}$$

$\overline{Mom_S}$ = average momentum change of the subjects

12. Returning to the impulse for each subject, calculate a subject specific force scale factor by dividing the average momentum change, step 11, by the impulse for each subject, step 3, multiplied by the time scale factor from step 10. Because the impulse is an integral of force over time, the product of the time scale factor and the force scale factor multiplied by the subject impulse is equivalent to the average momentum change.

$$\lambda_{Fj} = \frac{\overline{Mom_S}}{I_j \cdot \lambda_{Tj}}$$

λ_{Fj} = force scale factor for each subject

13. Calculate a subject specific scale factor for deflection by dividing the average deformation energy, step 5, by the subject deformation energy multiplied by the force scale factor from step 12. Because the

deformation energy is the integral of force over deflection, the product of the force scale factor and the deflection scale factor multiplied by the subject deformation energy is equivalent to the average deformation energy.

$$\lambda_{Dj} = \frac{\overline{E_D}}{E_{Dj} \cdot \lambda_{Fj}}$$

λ_{Dj} = deflection scale factor for each subject

14. Additional scale factors for the kinematic quantities of subject acceleration, velocity and displacement can be found from combining the scale factors for force and effective mass, λ_F/λ_m, to scale acceleration. Note that scaled velocity and displacement can then be obtained by numerical integration of the scaled acceleration; however, the time scale factor must be included as a multiplier in each integration.

A second deflection-deformation energy normalization method was also investigated that assumed the normalized maximum deflection of all of the subjects tested should be the same. Steps 12 and 13 above were modified as follows.

12a. Calculate a subject specific scale factor for deflection by averaging the maximum deflection of all subjects in the data set to obtain a standard maximum deflection and dividing this standard maximum deflection by each subject maximum deflection.

$$\lambda_{Dj} = \frac{\frac{1}{N}\sum_{j=1}^{N} D_{jmax}}{D_{jmax}}$$

13a. Calculate a subject specific scale factor for force by dividing the average deformation energy, step 5, by the subject deformation energy multiplied by the deflection scale factor from step 12a. Because the deformation energy is the integral of force over deflection, the product of the force scale factor and the deflection scale factor multiplied by the subject deformation energy is equivalent to the average deformation energy.

$$\lambda_{Fj} = \frac{\overline{E_D}}{E_{Dj} \cdot \lambda_{Dj}}$$

The principal assumption for developing typical response curve tolerance targets for a set of PMHS is the central mean theorem as cited above. A mean curve and plus and minus standard deviation tolerance bands can be constructed at each point in time. Note that after scaling the time, the response time histories for each subject will have to be re-sampled to a common time increment and the length in time will have to be set to that of the shortest time history in order to calculate the correct mean and standard deviations.

The two-dimensional standard deviation ellipse method [14] is used for developing a tolerance band for the force versus deflection curve because there is variation in both quantities. The procedure for generating the force versus deflection mean curve and standard deviation ellipse tolerance is:

1. The normalized time histories for deformation energy, force and deflection are generated for each subject using the subject specific scale factors from steps 5, 12 and 13 above (or 12a and 13a). Note that the three time histories for energy, force and deflection are parametrically related by the time base but that the deformation energy is not in a uniform increment of energy.
2. A new uniformly spaced deformation energy series is created by dividing the maximum of the mean deformation energy by a convenient length, say one thousand. Note that the smallest maximum energy should be used to calculate the mean deformation energy.
3. The set of force and deflection series are interpolated onto the uniformly spaced deformation energy series creating force and deflection series that are parametrically related by the uniform deformation energy series. Force and deflection versus deformation energy histories (rather than time histories) have now been created.
4. The means and standard deviations for force and deflection versus energy are then generated.
5. The force and deflection versus energy histories are then cross plotted in force versus deflection space along with the mean force versus deflection curve. The standard deviations for force and deflection at each energy point are used to create the standard deviation ellipse for force and deflection at each energy point [14]. The area of the ellipses are averaged and plotted at each point to form a statistically based, quantitative visual target (tolerance band) against which the performance of a dummy can be compared.

6. The original ellipse coefficient of variation is obtained by dividing one half of the area of the ellipse at each point by the product of the force and deflection [4] at that point to obtain a measure of variation that is analogous to the well-known coefficient of variation. The ellipse coefficients of variation at each point are then averaged to obtain an average coefficient of variation that represents the overall variation of the force versus deflection response in force versus deflection space. Because the coefficient of variation is very large when the mean is small, this calculation is only performed at points greater than 10% of the maximum of both force and deflection. Also, the ellipses are terminated at maximum deformation energy because it is only possible to interpolate force and deflection onto a uniform monotonically increasing energy series.

Finally it should be noted that the standard deviation ellipses are constructed from the standard deviations of force and deflection versus deformation energy curves in force versus deflection space. In previous studies [4, 14] the ellipses were constructed from the standard deviations of force and deflection versus time curves and then plotted in force versus deflection space which is misleading since the force versus deflection pairs of each subject based on the time are not co-located in force versus deflection space. The result of this phenomenon is that the time-based standard deviation ellipses do not represent the grouping of the force versus deflection curves in force versus deflection space. Interpolating force and deflection to a common deformation energy base resolves this problem; however, comparing the effectiveness of this deformation energy based normalization methodology to previous methods using the average ellipse coefficient of variation can only be done accurately if coefficients of variation are calculated the same way. In order to make a reasonable comparison among the several normalization methods, the coefficients of variation are calculated based on standard deviations calculated from the force and deflection time histories even though this is, strictly speaking, not correct. Since the deformation energy is not calculated for the previous methods, there is little choice. The coefficients of variation for the new methods are also calculated in force versus deflection space.

III. RESULTS

Rhule [14] presented force and deflection data from a set of 90° lateral and 60° antero-lateral oblique thoracic pendulum tests at a nominal 4.5 m/sec. The test conditions were the same for each subject. These data were used as an example data set in order to compare the performance of the various normalization methods. Of the five lateral tests and the four oblique tests, one each failed to meet the requirements of impulse and momentum and were not normalized leaving a four subject lateral subject set and a three subject oblique subject set.

A summary of the Rhule test conditions and the subject characteristics is presented in Table 1 [14]. Table 2 presents the modeled stiffness and damping coefficients and Figure 1 is an example of the lateral modeled response. Table 3 presents the deformation energy normalization scale factors.

Table 1. Subject Characteristic Data.

Test #	Impact Direction	Speed (m/s)	Gender	Age	Height (cm)	Body Mass* (kg)	Body Mass Index (kg/m^2)	Young -Adult T-Score	Chest Breadth (mm)	Chest Depth (mm)	Rib Cage Height (cm)	# Ribs Loaded by Impactor
802	90°	4.77	M	67	169	81.6	28.6	-0.2	314	233	31	4
902	90°	4.63	M	77	178	63.5	20	0.4	313.5	202.5	30	5
1001	90°	4.76	M	60	183	88.6	26.5	-0.2	360	209	32	4
1002	90°	4.44	M	29	175	78.5	25.6	1.7	274.5	212	31	5
Lateral			Avg	58	176.3	78.1	25.2	0.4	315.5	214.1	31.0	4.5
			Min	29	169	63.5	20	-0.2	274.5	202.5	30	4
			Max	77	183	88.6	28.6	1.7	360	233	32	5
803	60°	4.58	M	81	176.5	79.4	25.5	3.8	315	170	32	5
804	60°	4.47	M	81	175.3	92.5	30.1	0.1	342	180	30	4
1003	60°	4.56	M	48	175	78	25.5	0.1	336.5	205	31	4
Oblique			Avg	70	175.6	83.3	27.0	1.9	331.1	185	31	4.3
			Min	48	175	78	25.5	0.1	315	170	30	4
			Max	81	176.5	92.5	30.1	3.8	342	205	32	5

Table 2. Modeled stiffness and viscous constants for each subject.

Subject	k (N/m)	c (N/m/sec)
Lateral		
802	38,155	325
902	29,748	238
1001	45,344	344
1002	41,128	293
K mean	38,594	300
Oblique		
803	32,072	285
804	33,370	228
1003	43172	307
K mean	36,204	273

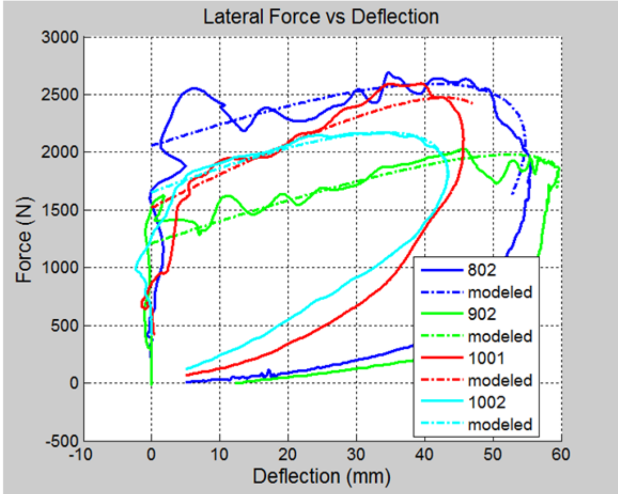


Figure 1. Plot of lateral force versus deflection data and the modeled response

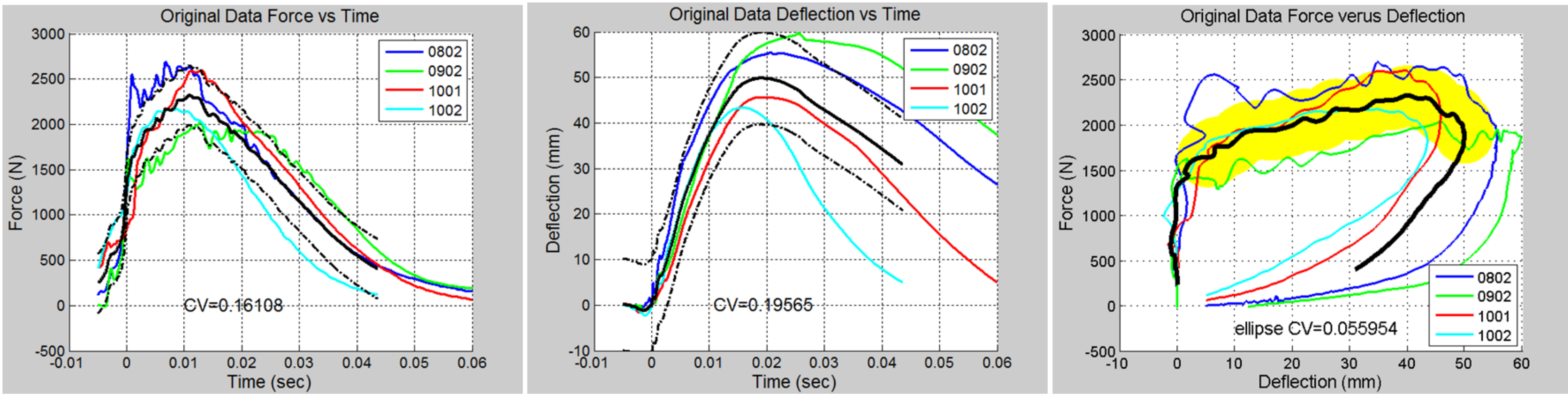
Table 3. Normalization factors

subject	λ_E deformation energy	λ_M effective mass	λ_F Force	λ_D Deflection	λ_T Time	λ_k Stiffness	λ_c Viscous
Lateral							
802	0.79	0.72	0.99	0.80	0.90	0.99	1.08
902	1.01	1.01	1.08	0.94	0.88	0.77	0.79
1001	1.09	1.24	0.90	1.21	1.14	1.17	1.15
1002	1.20	1.16	1.16	1.03	1.07	1.07	0.98
Oblique							
803	0.93	0.89	1.05	0.88	0.91	0.89	1.04
804	1.10	1.04	1.04	1.01	0.97	0.92	0.83
1003	1.03	1.09	0.87	1.19	1.11	1.19	1.12

Plots for the non-normalized data, the Eppinger method, the Mertz-Viano method, the Moorhouse method and the two deformation energy methods of this study are shown in Figures 2-7 for the lateral thoracic impact PMHS data set. In the force and deflection versus time plots, the mean and the mean plus and minus one standard deviation curves are also plotted. In the force versus deflection plots the standard deviation ellipses, parametrically related by time for all methodologies, are shown in yellow. Force versus deflection plots with the standard deviation ellipse targets calculated in force versus deflection space are shown in Figures 8 and 9 for the deformation energy and deflection-deformation energy normalization methods. The average ellipse coefficients of variation [4] are also shown on the force versus deflection plots. The average coefficients of variation based on time histories are also shown in Table 4 for ease of comparison and as calculated in force versus deflection space in Table 5. The same calculations are carried out for the oblique thoracic impact tests and presented in Tables 6 and 7.

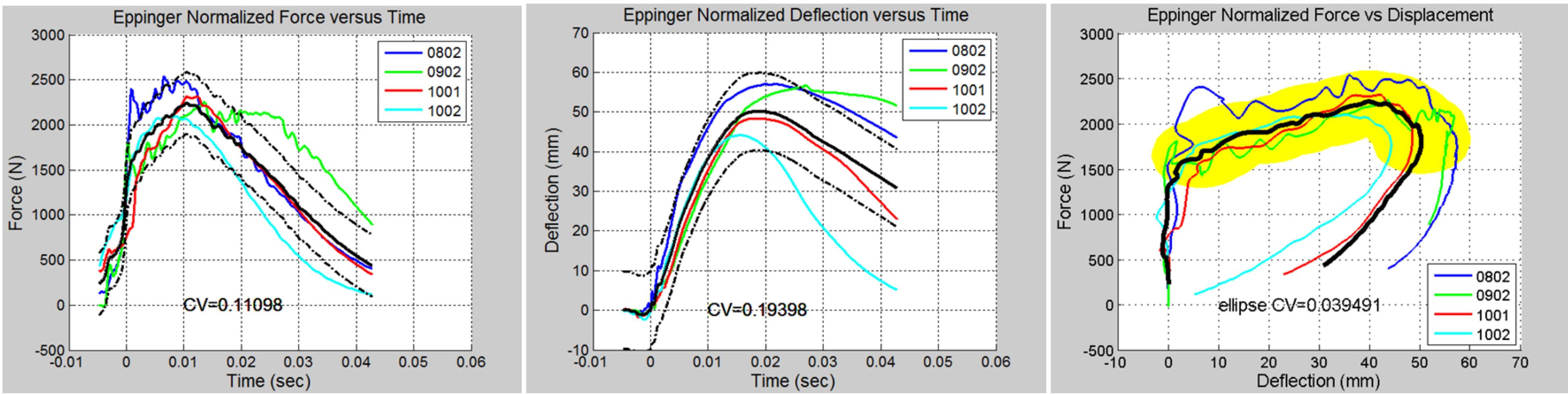
IV. Discussion

The objective of normalization is to develop scale factors, based on sound mechanical principles that are used to adjust the subject response to better represent a standard sized human, in this example the 50th percentile adult male. This deformation energy approach to normalization assumes that the subject response data can be used to normalize the response to the mean which is assumed to be a reasonable representation of the standard human (50th percentile male). Mertz was the first to suggest this response data approach with the effective mass approximation and Moorhouse also used it for effective stiffness. This study uses basic mechanical principles, conservation of energy and momentum, along with an assumption of linear



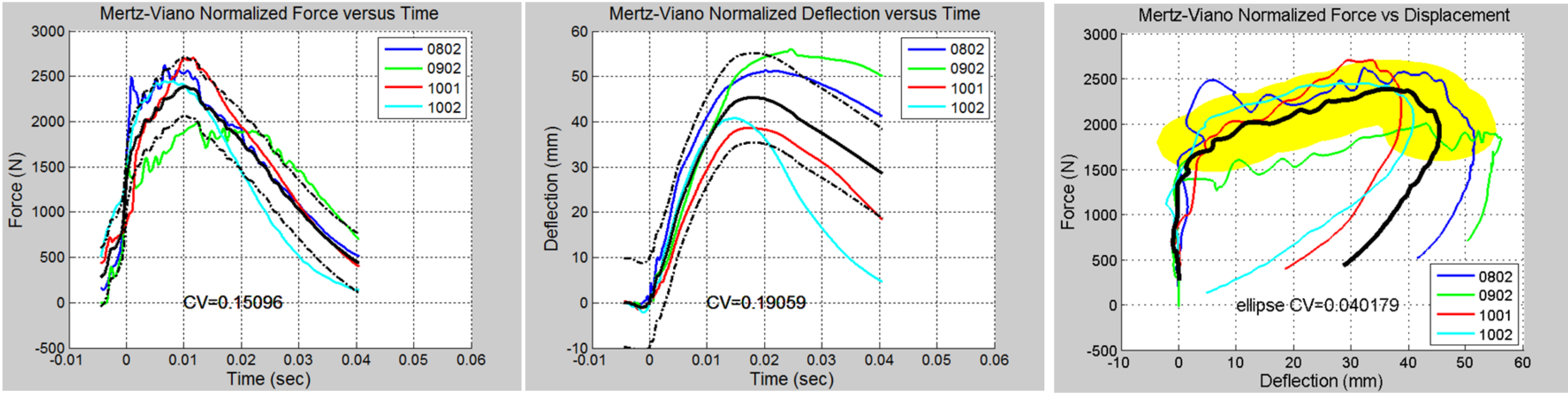
(a) (b) (c)

Figure 2. Non-normalized curves for the lateral thoracic impacts.



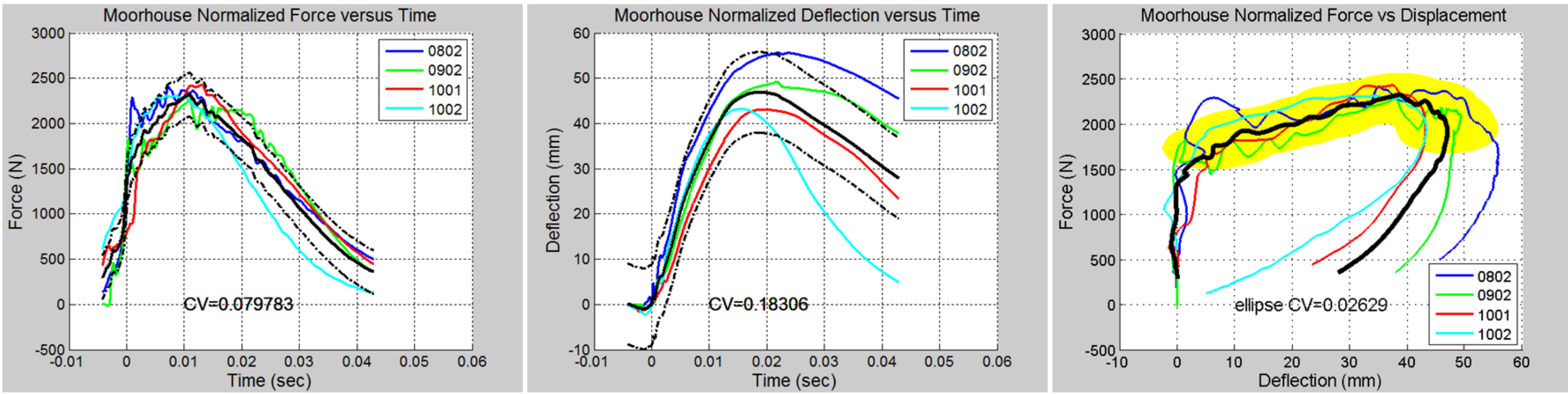
(a) (b) (c)

Figure 3. Eppinger mass normalization method for the lateral thoracic impacts.



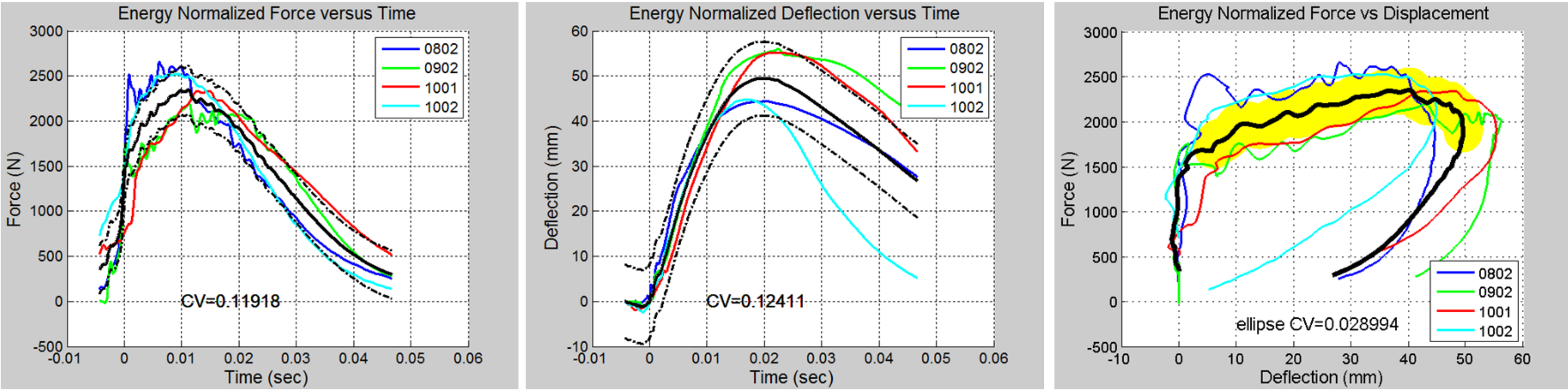
(a) (b) (c)

Figure 4. Mertz-Viano effective mass normalization method for the lateral thoracic impacts.



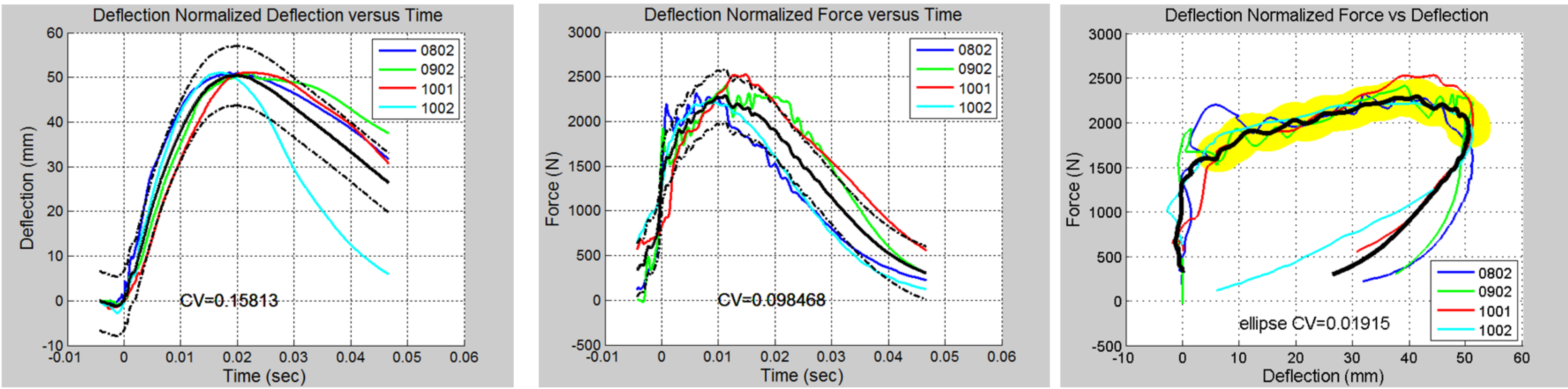
(a) (b) (c)

Figure 5. Moorhouse effective stiffness normalization method for the lateral thoracic impacts.



(a) (b) (c)

Figure 6. Deformation energy normalization method for the lateral thoracic impacts.



(a) (b) (c)

Figure 7. Deflection-deformation energy normalization method for the lateral thoracic impacts.

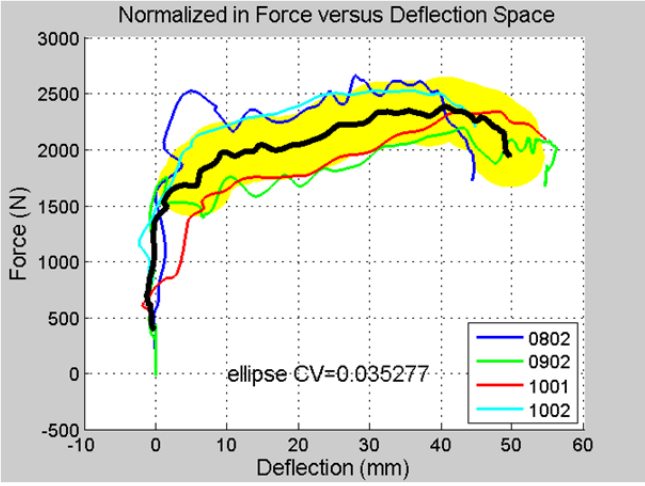


Figure 8. Force versus deflection for deformation energy normalization with standard deviation ellipses calculated in force deflection space.

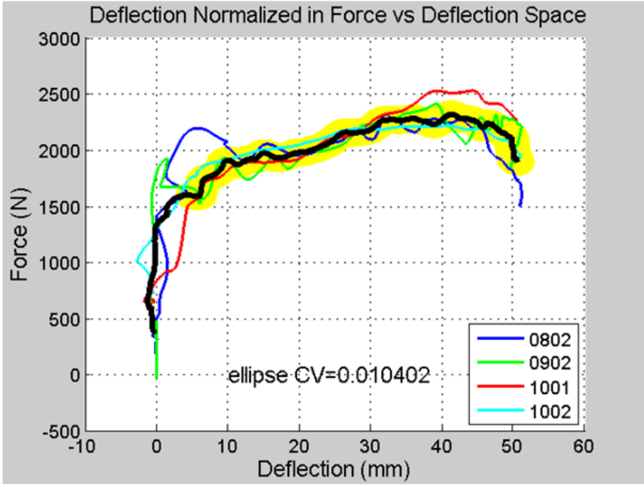


Figure 9. Force versus deflection for deflection -deformation energy normalization with standard deviation ellipses calculated in force deflection space

Table 4. Lateral tests- Comparison of average coefficients of variation calculated from time histories.

Method	Force vs Time	Deflection vs Time	Force vs Deflection
Non-normalized	0.1611	0.1957	0.0560
Eppinger Method	0.1110	0.1940	0.0395
Mertz-Viano Method	0.1469	0.1846	0.0518
Moorhouse Method	0.0798	0.1831	0.0263
Deformation Energy Method	0.1198	0.1241	0.0290
Deflection-Deformation Energy Method	0.0985	0.1581	0.0192

Table 5. Lateral tests - Comparison of average ellipse coefficients of variation calculated in force versus deflection space.

Method	Force vs Deflection
Deformation Energy Method	0.0353
Deflection-Deformation Energy Method	0.0104

Table 6. Oblique tests- Comparison of average coefficients of variation calculated from time histories.

Method	Force vs Time	Deflection vs Time	Force vs Deflection
Non-normalized	0.1611	0.1957	0.0560
Eppinger Method	0.1329	0.1384	0.0310
Mertz-Viano Method	0.0939	0.1112	0.0157
Moorhouse Method	0.0662	0.0743	0.0098
Deformation Energy Method	0.0875	0.0832	0.0132
Deflection-Deformation Energy Method	0.0932	0.0645	0.0083

Table 7. Oblique tests - Comparison of average ellipse coefficients of variation calculated in force versus deflection space.

Method	Force vs Deflection
Deformation Energy Method	0.0135
Deflection-Deformation Energy Method	0.0073

viscoelasticity to develop normalization factors that reduce the effect of human variation on response. The normalized responses are averaged and the standard deviations in time, or in force vs deflection space, provide a target against which a dummy response can be quantitatively compared when tested in the same way. Normalization is necessary for the design and development of a biofidelic dummy in order to focus the dummy response on accurately modeling the response of the standard sized human. An objective method for assessing biofidelity that quantifies the similarity of the dummy response to the standard human using the standard deviation as a metric is logical. The normalization factors modify the response data based on the subject anthropometry and dynamic response from tests. It is possible that reducing the variance in the response of the subject sample through normalization accounts for the human variation leaving largely experimental error represented by the standard deviation although this possibility is not examined in this study.

Eppinger postulated that PMHS anthropometric measures, such as mass, could be used to normalize PMHS response data to a standard size (e.g. 50th percentile male) for which the same anthropometric measure was known. This is an attractive concept and allows one to develop scale factors independently of the actual response data; however, the results presented here and elsewhere indicate that the method is only partially successful. Subject total body mass does not appear to predict structural response very well. Mertz utilized the response data from a PMHS and a single DOF linear elastic model system to develop scale factors for normalization of response data by using the force and velocity versus time histories to develop an effective mass for each PMHS. Viano improved the Mertz method for two DOF systems and Moorhouse improved the method further by using a linear elastic estimate of deformation energy to replace the use of characteristic length to develop the scale factor for stiffness. Each of these latter three investigators utilized the response data from each subject to develop scale factors to normalize the responses. This concept of using the response data to normalize the responses is now accepted by most researchers and is also used in this study.

The results presented are based on the data from a single study of thoracic pendulum impacts and are an example of the deformation energy approach. A feature of the approach presented in this study as well as the Mertz, Viano and Moorhouse studies, but not the Eppinger study, is that the response of a typical human to which the response data are being normalized is not known. The Eppinger method is based entirely on anthropometry and the standard sized 50th percentile male anthropometry is known. Mertz developed a methodology for using known anthropometry to relate the physical parameter of a subject to an unknown 50th percentile parameter, e.g., effective mass. The subject parameter of interest is divided by a relevant anthropometric measure that is known for that subject and also for the 50th percentile subject. The set of ratios obtained are then averaged and multiplied by the 50th percentile value of the anthropometric parameter to obtain an estimate of the typical value for the 50th percentile subject. This approach, which is also used in this study as well as in the Moorhouse study, provides a link to the typical sized subject but still assumes a normal distribution and the central mean theorem. Even if the data are normally distributed, sample sizes are usually very small and the assumption that a mean of the few sample time histories available is an accurate representation of the typical sized human is at best an approximation. Further, as future data become available from new PMHS studies using the same test condition, the mean response for the typical human should be adjusted. The essential flaw in the approach of using a relatively small sample to characterize a large population is obvious but until a large, well-defined sample becomes available the dependence on small data sets is unavoidable. The best known model of a human is currently the human cadaver and small sets of cadaveric response data provide the best biofidelity target for crash dummy response. The results presented here are an example of the deformation energy method. A truly representative thoracic force versus deflection biofidelity target should incorporate results from all available studies with well documented test conditions and response data.

Two related but somewhat different deformation energy methods are presented here for normalizing the response data. The first method utilizes deformation energy and a viscoelastic model to generate scale factors based on conservation of energy and conservation of momentum (impulse-momentum). The scale factor for time is obtained from modeling the event as a two-element viscoelastic model and the time scale factor is used in the impulse equation to obtain a scale factor for force which is then used, in turn, to obtain a scale factor in deflection using the equation for deformation energy. This approach is mechanistically sound, incorporates a viscous thoracic response and makes no additional assumptions. The second method makes the assumption that all subjects, when normalized, should have the same maximum deflection. This assumption causes the grouping of the normalized response curves to be much tighter. In this approach the maximum thoracic deflection of each subject in the data set is normalized to the average maximum deflection and the resulting scale factors for deflection are used with the deformation energy equation to obtain the scale factor for force. If the desire of the analyst is to obtain smaller biofidelity targets, i.e., smaller standard deviations and smaller ellipses, then this latter approach will provide those smaller targets. It must be recognized that unless one is confident that the typical maximum deflection obtained from averaging the maximum deflections from each subject is indeed a reliable estimate of the standard thoracic deflection and that all subjects should attain that maximum deflection (no natural variation), then the second approach may be artificially manipulating the response more than the first approach.

Because the scale factors for force and deflection that contribute to the normalized maximum deformation energy are indeterminate, a second condition is required to obtain those scale factors. Any reasonable and mechanistically sound condition can be used for this purpose. In the first approach a scale factor for time is derived for a viscoelastic model of the force versus deflection data (Appendix A) and impulse-momentum is used as the second condition to find a scale factor in force. In the second approach it is assumed that maximum thoracic deflection for a normalized subject is the average of the maximum deflections of the set of subjects and the averaged deformation energy is used directly to obtain a force scale factor for each subject. In the second approach the scale factor for time is also derived from the viscoelastic thorax model.

Relating force versus deflection parametrically through deformation energy permits the accurate construction of the standard deviation ellipses in force versus deflection space. The standard deviation ellipse is a statistical measure of the variation at each force deflection pair in two dimensions. This is perhaps the strongest reason to utilize the deformation energy method, or the deflection–deformation energy method, for normalizing the responses from a set of PMHS test subjects. The force versus deflection response of the standard, or typical, subject is extremely useful in the design and development of crash test dummies. Structural design is based on load and deformation rather than force versus time or deflection versus time and the dummy designer will find the force versus deflection biofidelity curve to be invaluable. Parametrically interpolating the force and deflection time histories onto a deformation energy base allows for an accurate representation of the variance in two dimensions in force versus deflection space.

The set of standard deviation ellipses at each point of the force versus deflection curve creates a target against which a dummy response can be quantitatively measured in force versus deflection space in the same way that time histories are evaluated using the Biofidelity Ranking System [17]. The ellipse target can also be used qualitatively in dummy design and development to ascertain how close the dummy response is to the PMHS data set.

A limitation of this methodology is the application to a single example of well-behaved thoracic impact responses of only four lateral and three oblique tests at one test configuration and impact speed. The method should be applied to additional data sets obtained from other test conditions with, preferably, more subjects included to thoroughly validate the approach.

V. CONCLUSIONS

Two variations for normalizing PMHS thoracic response data from a pendulum type test using the deformation energy and assuming a two DOF viscoelastic model of the thorax are presented. The use of deformation energy provides a consistent standard deviation calculation for the two-dimensional standard deviation ellipse tolerance on the force versus deflection response curve. The resulting normalized response data are qualitatively and quantitatively compared to previous methods for normalization. In most, but not all, cases the deformation energy methods provide less variation (tighter grouping of curves) than do the previous methods when compared using the average coefficient of variation.

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Appendix A. A solution for the elastic and viscous constants, k and c.

A two-element Kelvin-Voigt model can be used to estimate the elastic and viscous constants that best fit the measured force and deflection response time histories for each subject. Since we know the form of the equation for force as a function of deflection and deflection rate for the Kelvin-Voigt model and we have the force and deflection time histories, the constants can be estimated using a least squares approach without any other information.

Fit a 2^{nd} order polynomial to the deflection versus time data for each subject using any convenient least squares curve fitting algorithm (Matlab, Excel, etc.) to obtain the coefficients a_1 and a_2.

$$x(t) = a_2 t^2 + a_1 t$$
$$\dot{x}(t) = 2a_2 t + a_1$$

Write the equation for force in a two-element Kelvin-Voigt viscoelastic solid material as a function of deflection and deflection rate where *c* and *k* are the viscous and elastic constants.

$$f(t) = c\,\dot{x}(t) + k\,x(t)$$

Substitute for deflection and deflection rate from above

$$f(t) = c(2a_2 t + a_1) + k(a_2 t^2 + a_1 t)$$

$$f(t) = k a_2 t^2 + (k a_1 + 2c a_2)t + c a_1$$

Let $b_2 = ka_2$, $b_1 = ka_1 + 2ca_2$, and $b_0 = ca_1$. Writing b_1 in terms of b_2 and b_0 gives,

$$b_1 = \left(\frac{b_2}{a_2} a_1 + 2\frac{b_0}{a_1} a_2\right)$$

and,

$$f(t) = b_2 t^2 + \left(\frac{b_2}{a_2} a_1 + 2\frac{b_0}{a_1} a_2\right) t + b_0.$$

A least squares formulation can be written where *F(t)* is the applied force data for a subject.

$$Q = \sum [F(t) - f(t)]^2$$

$$\frac{dQ}{db_2} = 0 \qquad \frac{dQ}{db_0} = 0$$

These two equations and two unknowns can be solved for b_2 and b_0 and substituted to find k and c, where

$$k = \frac{b_2}{a_2} \qquad c = \frac{b_0}{a_1}$$