Optimized Phasing of PMHS Response Curves for Biofidelity Targets

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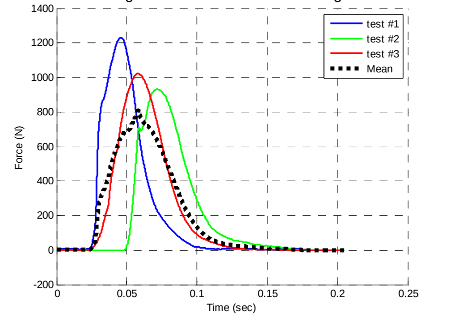
I.INTRODUCTION

When developing biofidelity targets (corridors) for evaluating a dummy, response curves from multiple post-mortem human subjects (PMHS) are often averaged [1]. Large phase differences due to large (obese) and small (thin) subjects can cause the average to be non-typical, Figure 1. The technique presented optimizes the phasing among all responses such that curve shapes can be optimally aligned and a typical average obtained.

Figure 1. Data Curves and Average



II. METHODS

The cross-correlations, T_{ij} , of all possible pairs of the response channel of interest from the subjects in the sample population are calculated and the lags, L_{ij}, that provide the maximum cross-correlation (the best phase fit) for each pair are tabulated [2]. Note that left shifts are positive lags and right shifts are negative lags. The sum of the lags for any set of response curves must equal zero. For example, given a triplet of response curves the lags

$$L_{12} + L_{23} + L_{31} = 0 \qquad (1)$$

because the maximizing lag of curve 2 with respect to curve 1 and the maximizing lag of curve 3 with respect to curve 2 completely defines the remaining lag of curve 1 with respect to curve 3. However, this may not be the maximizing lag for curves 1 and 3. Thus we seek the optimum set of lags that maximizes the cross-correlations of all three curves. This condition can be found for as many response curves as are in the sample population. For sample populations larger than three subjects all of the lag sum conditions are satisfied if all possible triplet conditions are satisfied because larger lag sums are linear combinations of the triplet sums. For four subjects

$$L_{12} + L_{23} + L_{31} = 0;\ L_{23} + L_{34} + L_{42} = 0;\ L_{34} + L_{41} + L_{13} = 0;\ L_{41} + L_{12} + L_{24} = 0. \qquad (2)$$

The objective function to be maximized is the sum of all cross-correlation permutations with the side conditions that all lag sum permutations equal zero. For four subjects the objective function would be

$$F = T_{12}(L_{12}) + T_{23}(L_{23}) + T_{34}(L_{34}) + T_{13}(L_{12}) + T_{24}(L_{24}) + T_{14}(L_{14}). \qquad (3)$$

Returning to our example of three subjects for simplicity, the Lagrange Multiplier technique [3,4] is used to create a set of linear equations that can be readily solved for the optimal lags among all subjects.

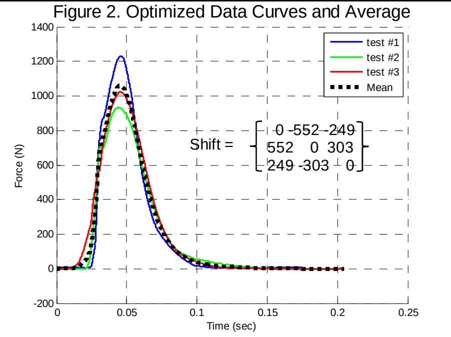
$$dF = \left(\frac{\partial F}{\partial L_{12}} + \lambda\right) dL_{12} + \left(\frac{\partial F}{\partial L_{23}} + \lambda\right) dL_{23} + \left(\frac{\partial F}{\partial L_{31}} + \lambda\right) dL_{31} = 0 \qquad (4)$$

The principle of independence provides three equations and the side condition, equation (1), is the fourth equation. Larger sample populations have more permutations and require more equations to solve for more unknown lags but the principle is the same.

III. RESULTS

This procedure provides a set of optimally aligned PMHS response curves that can be averaged to provide a meaningful typical response curve, Figure 2, against which a dummy response can be evaluated [5,6].

Figure 2. Optimized Data Curves and Average



IV. DISCUSSION AND CONCLUSIONS

A numerical procedure for optimizing the phasing among a set of any number of response time histories has been programmed in Matlab. The numerical method is based on an assumption that the first derivative of the cross-correlation function can be approximated by a straight line as it crosses zero. This approach provides a set of linear equations in the lags that can be readily solved. The size of the solution matrix will increase rapidly with the number of subjects in the sample population as the permutations increase. An adjusted time zero for the typical curve can be found by averaging the maximizing lags.

V. REFERENCES

1. Maltese, Stapp 2002; 2. Cooper, Methods of Signal and System Analysis; 3. Kaplan, Advanced Calculus; 4. Sokolnikoff, Mathematics of Physics and Modern Engineering; 5. Rhule, Stapp 2003; 6. Rhule, ESV 2011.

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