Optimal Cutoff Frequencies for Filtering Linear Acceleration and Angular Velocity Signals Associated with Laboratory Head Impacts Measured with Externally Mounted Sensors

Ryan Gellner, Mark Begonia, Susanna Gagliardi, Gregory Tierney, Steve Rowson

Abstract Kinematic measurements made at the head centre of gravity (CG) are most often filtered using SAE J211 recommendations, but there is little guidance for measurements taken at locations other than the CG. This study introduces a new method for determining optimal kinematic filtering. We then use this method to establish optimal cutoff frequencies for lowpass filters applied to six-degree-of-freedom (6DOF) signals when measured external to the CG. We conducted laboratory head impact tests using a pendulum impactor striking a bare headform to emulate impact scenarios observed in on-field environments. Impact conditions included four impact energies, two impact durations, and four impact locations. We collected triaxial linear accelerations at the headform's CG and triaxial linear accelerations and angular velocities at the teeth. We compared measurements at the teeth to ground truth determined through rigid body dynamics equations of motion. Teeth measurements were filtered with over 100,000 combinations of linear acceleration and angular velocity cutoff frequencies. Mean bias and variance were both considered when quantifying error. A range of suitable filter combinations was identified to minimise bias and variance, with the recommended filter combination being 300 Hz for linear acceleration and 300 Hz for angular velocity (CFC 180).

Keywords Acceleration transformation, head impact, filtering, cutoff frequency, error minimisation

I. INTRODUCTION

Head kinematics are often measured external to the centre of gravity (CG) of the head [1,2]. Most head injury metrics and filter recommendations assume measurements made at the CG [3]. To transform linear acceleration from external measurements to the CG, we need accurate measurements of both linear acceleration at the external measurement location and accurate measurements of angular velocity and angular acceleration of the head.

The accuracy of angular velocity measurements is not affected by impact-relevant filter cutoff frequencies because dominant frequencies in velocity measurements are generally low. However, when differentiating angular velocity to compute angular acceleration, the cutoff frequency can affect accuracy because differentiation increasingly amplifies high-frequency content in the signal. Furthermore, the correct value of angular acceleration is unknown because the frequency content is altered by the necessary step of differentiation in a six-degree-of-freedom (6DOF) gyroscope instrumentation package or by calculation in a nine-accelerometer array [4].

Here we introduce a new method to determine angular and linear kinematic measurement truth using a 6DOF instrumentation package. Angular measurements must be accurate to complete an accurate transformation of linear acceleration to the CG of a head. This approach exploits the rigid body dynamics motion equations to combine angular and linear kinematics into transformed linear acceleration and compares it to linear acceleration measured at the CG. CG measurements are assumed to be ground truth. We then use this method to identify optimal cutoff frequencies for gyroscopes and linear accelerometers in external laboratory 6DOF measurement packages to be consistent with CFC 1000-filtered accelerometer signals at the CG.

II. METHODS

Data Collection

We conducted laboratory head impact tests using a pendulum impactor and a range of impactor faces, from

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soft padding to rigid nylon, against a medium National Operating Committee on Standards for Athletic Equipment (NOCSAE) headform. These tests generated field-relevant impact durations. Data were filtered using a fourth-order phaseless lowpass Butterworth filter at a range of cutoff frequencies from 5 Hz to 1650 Hz. Peak transformed linear acceleration output by each cutoff frequency combination was then compared to peak linear acceleration which had been measured at the CG and filtered using the SAE J211 recommendation (CFC 1000, 1650 Hz cutoff frequency). The resulting mean bias and variance of error were used to rank the cutoff frequency combinations.

The gravity-driven pendulum impacted a bare medium NOCSAE headform mounted on a Hybrid III neck [5] atop a 5-degree-of-freedom linear slide table. The headform was modified so that 3D-printed dental arches could be mounted in an anatomically correct location for mouthguard testing [6]. For this test series, the 3D-printed dental arches were removed and replaced with a custom plate, which allowed us to mount laboratory instrumentation at the teeth location (Fig. 1). The dentition measurement package was positioned 57.4 mm anteriorly and 63.1 mm inferiorly relative to the headform CG.

The headform was instrumented with three linear accelerometers (Endevco 7264b-2000) and three angular rate sensors (DTS ARS3 Pro 18k) at the teeth and three linear accelerometers (Endevco 7264b-2000) at the CG. Signals measured from these devices included 50 ms of pre-trigger and 100 ms of post-trigger samples. Data collection for all instrumentation was triggered simultaneously when the dominant axis exceeded 5 *g* at the CG (e.g., the x-axis for impacts to the front location). The sampling rate of the instrumentation was 20 kHz, and a hardware anti-aliasing filter with a cutoff frequency of 4 kHz was applied to all channels before any other operations.

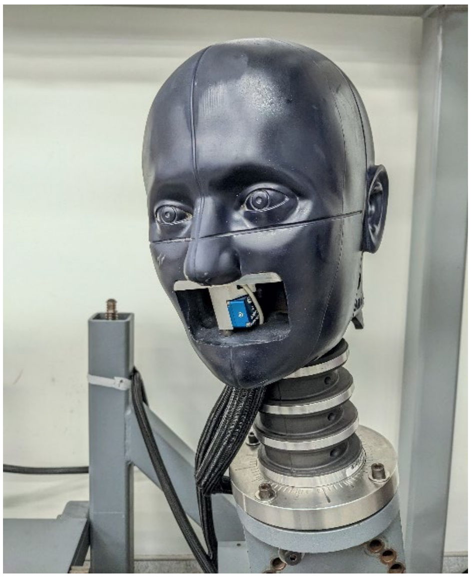

Fig. 1. A medium NOCSAE headform was instrumented with triaxial linear accelerometers and triaxial angular rate sensors at the teeth and triaxial linear accelerometers at the CG. A custom plate was developed to attach the laboratory-grade instrumentation at the teeth location.

Two faces covered the pendulum impactor: a rigid nylon face or a vinyl nitrile foam pad (VN1000, Dertex Corporation) (Fig. 2). These faces generated different linear acceleration durations: approximately 3 ms (rigid) and 7 ms (VN1000). Four locations were impacted on the headform (Rear, Rearboss, Frontboss, and Front) at four different pendulum angles, as described in detail in previous studies [7]. These four pendulum angles were representative of target impact severities of 25 *g*, 50 *g*, 75 *g*, and 100 *g*. Peak linear acceleration targets were said to have been achieved if the impact resulted in a peak resultant linear acceleration of plus or minus 5% of the target linear acceleration. Each configuration of location, severity, and impactor face underwent two trials; the padded tests were repeated to collect two padded impacts per rigid condition for a total of 96 tests.

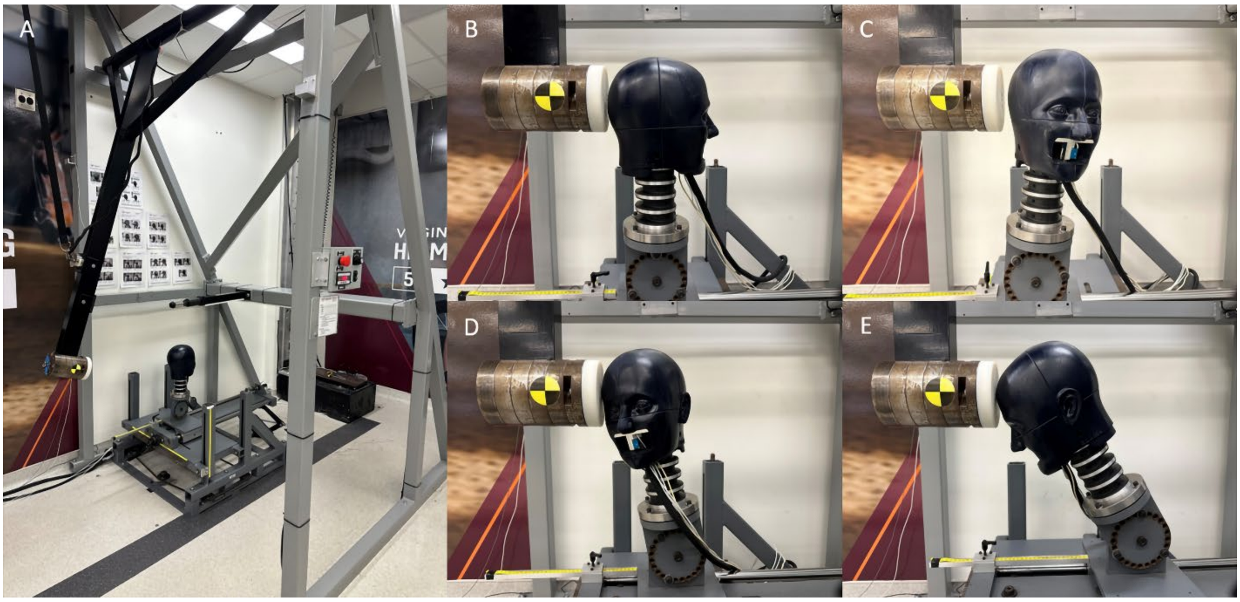

Fig. 2. A pendulum impactor (A) accelerated a bare NOCSAE headform. Four locations on the head were hit: Rear (B), Rearboss (C), Frontboss (D), and Front (E). Impactor faces were rigid nylon or VN1000, representing impact durations of 3 ms and7 ms, respectively. Four target linear accelerations were used to represent a range of impact severities: 25 *g*, 50 *g*, 75 *g*, and 100 *g*.

Theoretical Basis for Kinematic Ground Truth

Equations of dynamic motion for a rigid body enable the transformation of linear acceleration from one point (P) on a rigid body to another point (CG) on the same body so long as the body's angular velocity and angular acceleration are known (Equation 1 and Fig. 3). Thus, the optimal cutoff frequency choices for linear acceleration and angular velocity measurements at point P are those which minimise the error in the transformation equation output when compared to linear acceleration measured directly at point CG. These cutoff frequencies are optimal because they drive the system to obey the laws of motion most accurately.

$$\overrightarrow{a_{CG}} = \overrightarrow{a_P} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{\alpha} \times \vec{r} \quad (1)$$

Where a_i is linear acceleration at point *i*, r is the vector distance from point CG to point P, ω is angular velocity, and α is angular acceleration of the rigid body.

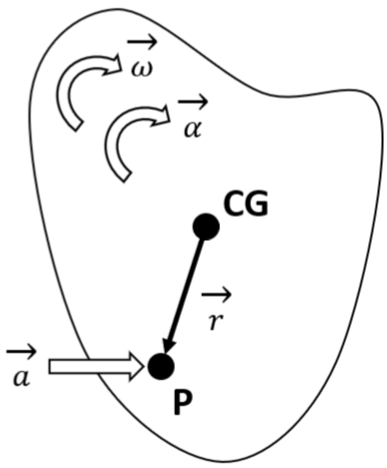

Fig. 3. An exemplar rigid body undergoing dynamic motion. Linear acceleration of one point on a rigid body can be transformed to any other point on the same body given knowledge of the vector distance between the two points and the body's angular velocity and angular acceleration.

The goal of this approach was to use an equation (here, the transformation of linear acceleration) grounded in physical truth, which enabled the choice of optimal cutoff frequencies for linear and angular measures simultaneously. The transformation of linear acceleration from one point on a rigid body to another fit this requirement because it included the required variables. The output of transformed linear acceleration inherently includes the error from angular velocity, angular acceleration, and linear acceleration. This approach requires angular velocity, angular acceleration, and linear acceleration curves to align with truth. Transformed linear acceleration would be erroneous if any of these curves were off from truth. Our analysis considered the entire time histories of the kinematic signals. However, we used peak values of the transformed linear acceleration curves to characterize transformation error because the curves' peaks are most sensitive to cutoff frequency. Without this transformation approach, comparing two measures of angular velocity or angular acceleration makes the choice of ground truth unclear because all points on a rigid body undergo the same angular motion in theory.

Data Processing

Linear acceleration and angular velocity curves collected at the teeth were filtered in sequential combination using a 4th-order lowpass, phaseless digital Butterworth filter across a range of 330 cutoff frequencies (-3 dB points), from 5 Hz to 1650 Hz in 5 Hz increments, for a total of 108,900 filter combinations. This filter was functionally equivalent to an SAE J211 filter and fell within the corridors for digital filters specified in that document (Fig. A3, Appendix). We then filtered headform CG linear acceleration with a cutoff frequency of 1650 Hz, equivalent to CFC 1000. Angular acceleration was calculated through five-point stencil differentiation of filtered angular velocity, as recommended in [8].

We rotated the CG and the dentition measurement packages to match SAE J211 coordinate axes definitions, requiring the using a Y-axis rotation matrix after filtering. The signals collected at the CG were at -20 degrees about the Y-axis, and those collected at the dentition were at +12 degrees about the Y-axis. We then removed bias by subtracting the average value of the first 7.5 ms of pre-trigger data from the entire signal.

We computed error as the difference between the peak values of the transformation equation output (transformed linear acceleration) and the CG linear acceleration measurement. Peak values occurred at nearly identical times for both transformed and CG curves (on average one sample difference), the difference in timing being only 0.05 +/- 0.32 ms (mean +/- sd) across all tests. Percent error was calculated by dividing the CG measurement into the computed error (Equation 2). Percent errors were averaged across all tests for each linear-angular cutoff frequency combination to quantify mean bias. The standard deviation of percent error for each combination was also calculated to quantify scatter. We used these two measures' root sum of squares (RSS) as the minimisation objective for determining optimal and acceptable minimum cutoff frequency combinations for accurate measurements at the teeth location.

$$\epsilon_{\%} = \frac{PLA_T - PLA_{CG}}{PLA_{CG}} \times 100\% \quad (2)$$

Where $\epsilon_{\%}$ is percent error, PLA_T is peak linear acceleration from the transformed teeth measurement, and PLA_{CG} is peak linear acceleration from the measurement at the CG.

We completed all data post-processing in MATLAB R2022a (MathWorks – Natwick, MA). RStudio 2022.07.2 (R Foundation for Statistical Computing, Vienna, Austria) was used to calculate error and generate visualisations.

III. RESULTS

The peak linear acceleration values collected in these tests were 62.5 ± 28.2 *g* (mean ± standard deviation) and had a range from 23.6 *g* to 108.1 *g*. Acquired peak angular velocity measurements were 12.5 ± 5.62 rad/s and ranged from 3.47 rad/s to 23.2 rad/s. Angular acceleration curves generated peak values of 3495 ± 1948 rad/s^2 ranging from 1032 rad/s^2 to 9351 rad/s^2.

The optimal cutoff frequency combination for measurements made at the teeth location and transformed to the head CG was 320 Hz for linear acceleration and 300 Hz for angular velocity. This combination had an RSS error of 4.36%. The cutoff frequency combination with the lowest mean bias was 1430 Hz for linear acceleration and 80 Hz for angular velocity. This combination had 0% mean bias but an RSS error of 16%. The cutoff frequency combination with the least scatter was 360 Hz for linear acceleration and 335 Hz for angular velocity. This combination had an RSS error of 4.60%. A wide range of cutoff frequency combinations had near-zero mean percent error. Both linear and angular cutoff frequencies influenced mean bias, but the angular cutoff frequency influenced the scatter substantially more than the linear cutoff frequency (Figs A1 and A2, Appendix). Table I details noteworthy cutoff frequency combinations.

TABLE I
PERCENT ERROR IN TRANSFORMED PEAK RESULTANT LINEAR ACCELERATION

Importance of Row	Linear Acceleration Filter Cutoff (Hz)	Angular Velocity Filter Cutoff (Hz)	Mean % Error (μ_e)	Std Dev % Error (σ_e)	RSS % ($\sqrt{\mu_e^2+\sigma_e^2}$)
Minimum Resultant	320	300	0.88	4.27	4.36
Minimum Mean	1430	80	6.55e-4	16.2	16.2
Minimum Std. Dev.	360	335	1.99	4.15	4.60
SAE J211	1650	300	5.74	6.63	8.77
CFC 180	300	300	0.43	4.43	4.46
Closest to 0/0 Hz on 5% Line	285	245	-0.82	4.94	5.00
Closest to 0/0 Hz on 10% Line	220	160	-6.02	8.06	10.1

A range of cutoff frequency combinations limited RSS error to 5% or less (Fig. 4). The 5% ellipse included cutoff frequencies of approximately 255 Hz to 400 Hz for linear acceleration signals and 240 Hz to 380 Hz for angular velocity signals. The lowest cutoff frequency combination to remain at 5% RSS error was 285 Hz for linear acceleration and 245 Hz for angular velocity. To limit RSS error to less than 10%, the linear acceleration cutoff frequency had a minimum (approximately 185 Hz) but no cap before 1650 Hz, but the angular velocity cutoff frequency had to remain between about 210 Hz and 590 Hz. The lowest cutoff frequency combination to stay at 10% RSS error was 220 Hz for linear acceleration and 160 Hz for angular velocity.

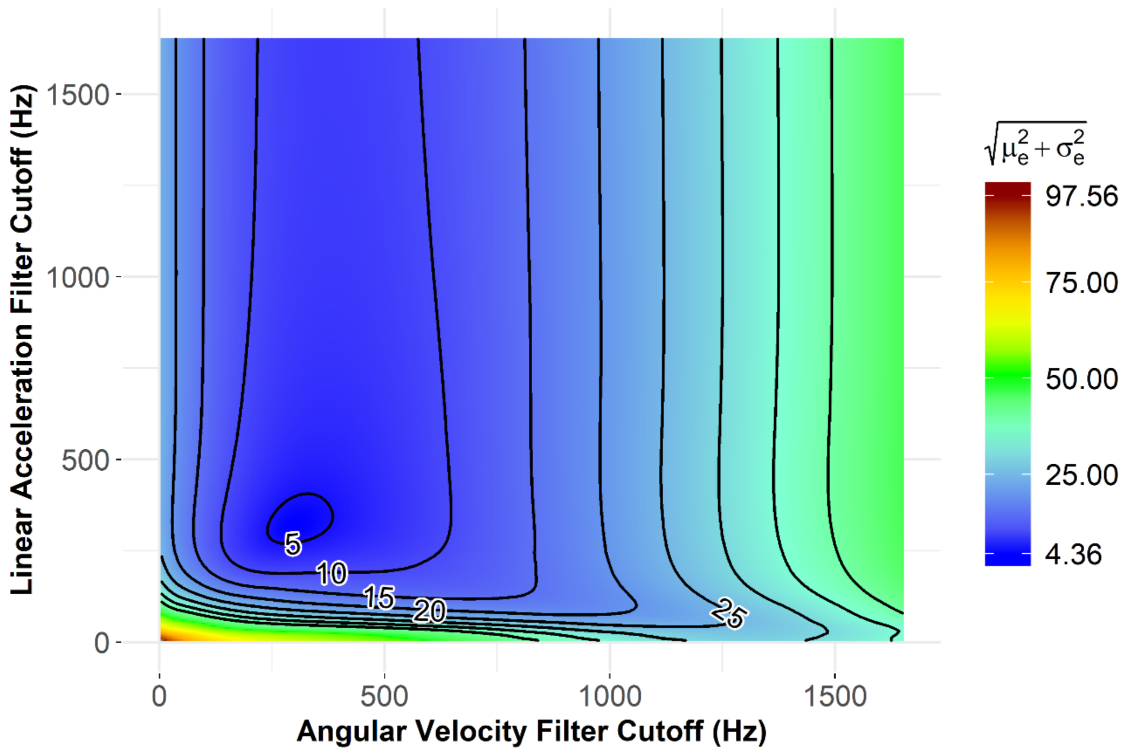

Fig. 4. RSS error of transformed peak linear accelerations by cutoff frequency combination. Contour lines represent 5% RSS error spacing; the colour bar runs from minimum to maximum resultant errors in the dataset.

IV. DISCUSSION

Since its publication, the SAE J211 document has been the "gold source" for much of the injury biomechanics field. The filter classes this document prescribes enable consistent and meaningful measurements to be made and compared across laboratories and studies. Linear acceleration at the head CG is designated a CFC 1000 filter, equivalent to a 1650 Hz cutoff frequency; angular velocity at the head CG is designated a CFC 180 filter, equal to a 300 Hz cutoff frequency. Little work has been done to quantify the effects of these cutoff frequencies on measurements made at locations other than the head CG. This study sought to optimize cutoff frequency choice for a 6DOF measurement package in an idealized laboratory environment when measurements were taken at locations away from the CG.

Since the measurement packages at both locations in our study were rigidly attached to the headform, it begs the question: why was a more aggressive filter needed for linear acceleration measured at the teeth to match the linear acceleration measurements at the CG? We know the transformation process introduces additional noise to the output signal. Another possible reason was a breakdown in the assumption of rigidity of the headform in severe rigid impacts. In this case of rigidity breakdown, the two measurement locations would have experienced different frequency content. We explored both potential reasons for needing a lower cutoff frequency at the teeth by plotting power spectral density (PSD) for the unfiltered linear acceleration of the dominant X axis in the 100 *g* frontal rigid impact as an exemplar case (Fig. 5).

Linear acceleration that had been transformed (green line) matched the CG measurement PSD (gray line) well at low frequencies but exhibited higher noise content past approximately 450 Hz. Noise also increased rather than decreased past this point because high-frequency content was introduced when we combined angular acceleration with linear acceleration during transformation. Differentiation of angular velocity inherently amplifies high-frequency noise. In the same impact, the untransformed linear acceleration signal at the teeth (black line) along the longitudinal axis overtook the CG signal and exhibited greater noise past 450 Hz. The authors, therefore, suggest that both effects may play a role. Because the untransformed signal was different than the CG signal, instrumentation at the teeth may experience different frequencies than the instrumentation at the CG. However, transforming linear acceleration also clearly introduces additional high-frequency noise to the final outputted linear acceleration signal transformed to the CG. Thus, the combination of these effects necessitates a lower cutoff frequency for linear acceleration at the teeth than at the CG to enable the two signals to best match in their peak values.

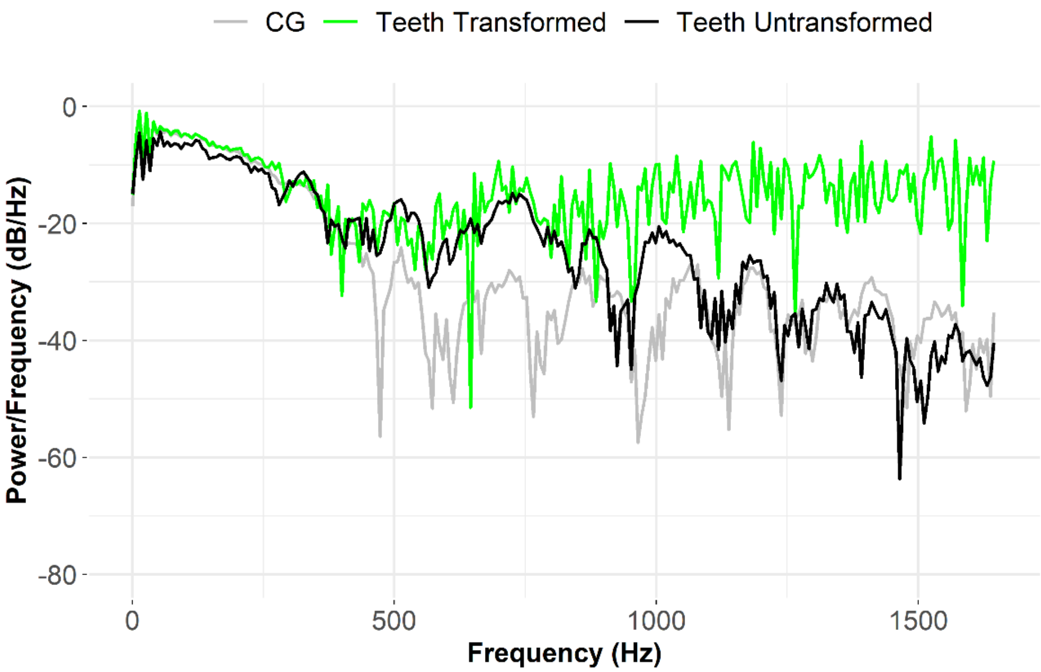

Fig. 5. A comparison of the power spectral density (PSD) of unfiltered linear acceleration measurements in the 100 *g* Front Rigid impact condition. Transformed linear acceleration exhibits greater noise at high frequencies due to the transformation process. The two untransformed accelerometers (black – teeth, gray – CG) also appear to experience different frequency content, particularly past 450 Hz.

Other studies have attempted to find a similar threshold for error reduction in post-processing techniques and have produced similar results. Wu *et al.* described minimum recommended bandwidths for lowpass filters for linear acceleration and angular velocity measurements [1]. Their study similarly noted that the limiting cases were high-frequency rigid impacts and found the need for a minimum 500 Hz bandwidth for error reduction below 10% in linear acceleration-based injury measures. The 500 Hz accelerometer cutoff point found in their study fits within the 10% RSS bounds in Fig. 4. However, they used full bandwidth signals as ground truth, as opposed to the SAE J211-filtered signals used in this study as ground truth, which may explain the slightly higher frequency recommended in their study. Cobb *et al.* sought an optimal filter for angular rate sensors [9]. That study identified CFC 175 (cutoff frequency: 292 Hz) as optimal for matching angular acceleration computed using a nine-accelerometer array package in the headform. The optimal cutoff frequency for angular rate sensors in that study is very similar to the cutoff frequency for angular velocity identified here and would fall within the 5% ellipse for RSS error.

Another study by Newman *et al.* set out to compare linear and angular accelerometer instrumentation errors from different measurement techniques and associated post-processing (a 3-2-2-2 linear accelerometer array, an angular accelerometer package, and a 2D in-line linear accelerometer array) [10]. Their study explored the effects of filtering linear accelerometers at CFC 180 when calculating angular acceleration "in order to help minimize the influence of spurious mechanical noise on the angular acceleration calculations." They found that CFC 180 generated approximately 8% linear and 6% angular acceleration errors in low-speed, unhelmeted (rigid) head impacts. Their study measured angular and linear acceleration directly but did not use rigid body transformation and did not measure angular velocity directly. As noted in Newman's study, "filtering effects depend on the system considered." As such, the frequency cutoff combinations denoted in the present study are valid for the presented system of a six-degree-of-freedom measurement package, including three linear accelerometers and three gyroscopes located away from the CG of the head. Other systems may have different optimal cutoff frequencies.

This study provides 5% and 10% error lines for measurements made with laboratory-grade instrumentation in an idealized environment. These error lines may be a starting point for similar studies involving instrumented mouthguards, which will likely require lower cutoff frequencies due to their inherently greater noise environment. Researchers should use measurements collected from field devices to repeat the described process and determine optimal filtering for linear acceleration and angular velocity signals recorded by those devices.

Limitations of this study include that rigid impacts were present in this dataset at a ratio of 1:2 compared to padded impacts. This ratio may not represent the rate at which impacts to hard surfaces occur in the field; however, rigid impacts are the limiting case for this study due to their higher frequency content, and they do happen in the field [11]. Second, the NOCSAE headform used for this study may have a different frequency response than a human head or other head forms, and the use of a different headform may result in different optimal filter choices. However, the NOCSAE head form represents a human head in shape and size [22], and researchers have successfully applied SAE J211 filter recommendations to various head forms and head sizes for years. Thus, we expect limited differences in the presented results across head forms. Third, using another differentiation technique could result in different optimal filter cutoff frequency choices because differentiation techniques may uniquely amplify noise. The five-point stencil differentiation technique used in this study is recommended by SAE J1727 [3,8]; hence, we felt it was the best choice to calculate angular acceleration from angular rate data. Fourth, the instrumentation in this study was laboratory grade; if this study were repeated with lower-quality sensors, the optimal filter cutoff choices would likely differ. Finally, error was computed as the difference in peak resultant linear accelerations because head impact studies commonly report peak values to define impact severity and because this highlighted sensitivity to filter cutoff frequency. Other methods of determining error between signals, such as root mean square error (RMSE), may result in slightly different optimal filter combinations.

V. CONCLUSIONS

A common choice of cutoff frequencies for instrumentation located external to the CG in laboratory and real-world environments would benefit the head impact biomechanics field. Optimal filter combinations for use in collecting and reporting 6DOF head kinematics at external locations on the head will allow future studies to produce generalisable data comparable to historical data. Here, we have described a method for determining optimal cutoff frequencies for linear acceleration and angular velocity measurements by relying on rigid body dynamics. This method can be repeated with other sensors, such as instrumented mouthguards, to determine optimal cutoff frequencies for accurate measurement with those sensors. We recommend a cutoff frequency of 300 Hz (CFC 180) for linear acceleration and angular velocity signals when taking measurements at the external locations on the head using laboratory-grade sensors.

VI. Acknowledgement

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VII. References

[1] Wu, L.C., Laksari, K., et al. (2016) Bandwidth and sample rate requirements for wearable head impact sensors. *Journal of biomechanics*. **49**(13): pp. 2918-2924
[2] Funk, J.R., Cormier, J.M., et al. (2011) Head and neck loading in everyday and vigorous activities. *Ann Biomed Eng*. **39**(2): pp. 766-76
[3] SAE. (2022) Instrumentation for Impact Test Part 1 - Electronic Instrumentation J211/1_202208.
[4] Padgaonkar, A.J., Kreiger, K.W., and King, A.I. (1975) Measurement of Angular Acceleration of a Rigid Body Using Linear Accelerometers. *J Appl Mech*. **42**: pp. 552-556
[5] Cobb, B.R., Zadnik, A.M., and Rowson, S. (2016) Comparative analysis of helmeted impact response of Hybrid III and National Operating Committee on Standards for Athletic Equipment headforms. *Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology*. **230**(1): pp. 50-60
[6] Kieffer, E.E., Begonia, M.T., Tyson, A.M., and Rowson, S. (2020) A two-phased approach to quantifying head impact sensor accuracy: in-laboratory and on-field assessments. *Annals of biomedical engineering*. **48**: pp. 2613-2625
[7] Tyson, A.M., Duma, S.M., and Rowson, S. (2018) Laboratory evaluation of low-cost wearable sensors for measuring head impacts in sports. *Journal of applied biomechanics*. **34**(4): pp. 320-326
[8] SAE. (2015) Calculation Guidelines for Impact Testing J1727_201502.
[9] Cobb, B.R., Tyson, A.M., and Rowson, S. (2018) Head acceleration measurement techniques: reliability of angular rate sensor data in helmeted impact testing. *Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology*. **232**(2): pp. 176-181
[10] Newman, J.A., Beusenberg, M.C., Shewchenko, N., Withnall, C., and Fournier, E. (2005) Verification of biomechanical methods employed in a comprehensive study of mild traumatic brain injury and the effectiveness of American football helmets. *Journal of biomechanics*. **38**(7): pp. 1469-81
[11] Kieffer, E.E., Vaillancourt, C., Brolinson, P.G., and Rowson, S. Using in-mouth sensors to measure head kinematics in rugby. *Proceedings of IRCOBI conference*, 2020 Online.
[12] Winter, D.A., (2009) "Biomechanics and motor control of human movement". *John Wiley & Sons*, 2009.
[13] Selling, J. "Multi-pass Filter Cutoff Correction" Internet https://www.codeproject.com/Articles/1267916/Multi-pass-Filter-Cutoff-Correction. [cited 2022 August].

VIII. Appendix

A. Percent Error Quantified by Mean and Standard Deviation

A narrow band of cutoff frequency combinations, decreasing in width from left to right as angular velocity cutoff frequency increases, enabled optimal (-1% to 1%) filtering for matching transformed linear acceleration to the headform CG peak measurements on average (Fig. A1). Both linear and angular cutoff frequencies affected mean bias. A transition can also be seen from underpredicting to overpredicting once filter combinations do not filter aggressively enough (blue regions). Overpredicting was generally of lower percent error than the most egregious cases of underpredicting. Cutoff frequency combinations in the very low-frequency range have their average error increase rapidly.

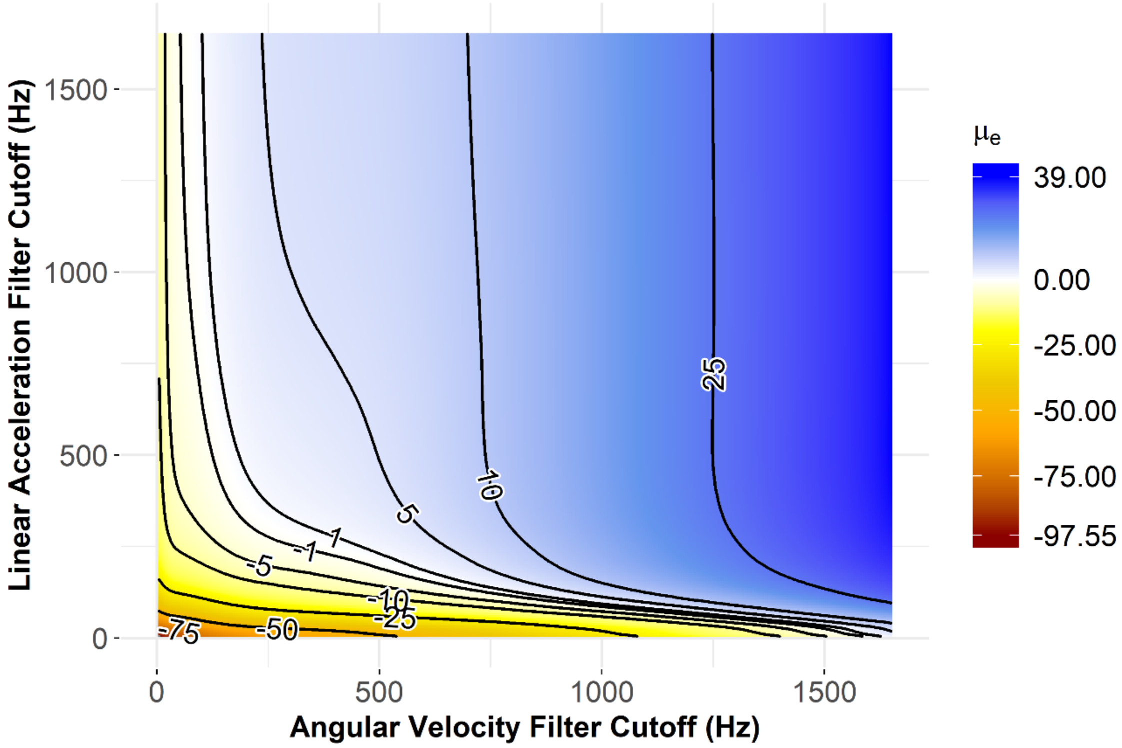

Fig. A1. Mean percent error of transformed peak linear acceleration by cutoff frequency combination. Contour lines represent unequal spacing, with the bottom left corner line being over 75% absolute mean error; the colour bar runs from minimum to maximum mean errors found in the dataset.

The lowest variation in error was found between 250 Hz and 500 Hz filter cutoff frequency for angular velocity in the 5% contour region (Fig. A2). The error standard deviation was much more sensitive to angular velocity cutoff frequency than linear acceleration cutoff frequency. The variation was similar for all linear acceleration filter cutoffs above approximately 300 Hz, except within the 5% contour line, which had its cap at about 600 Hz linear acceleration. The lowest standard deviation (0.99%) occurred in the bottom left corner of this plot. Disregarding points in that region due to their high mean error (Figure A1, above), we find that the point with the lowest standard deviation is 360 Hz cutoff for linear acceleration and 335 Hz for angular velocity, which lives in the middle of the plot, at 4.15%.

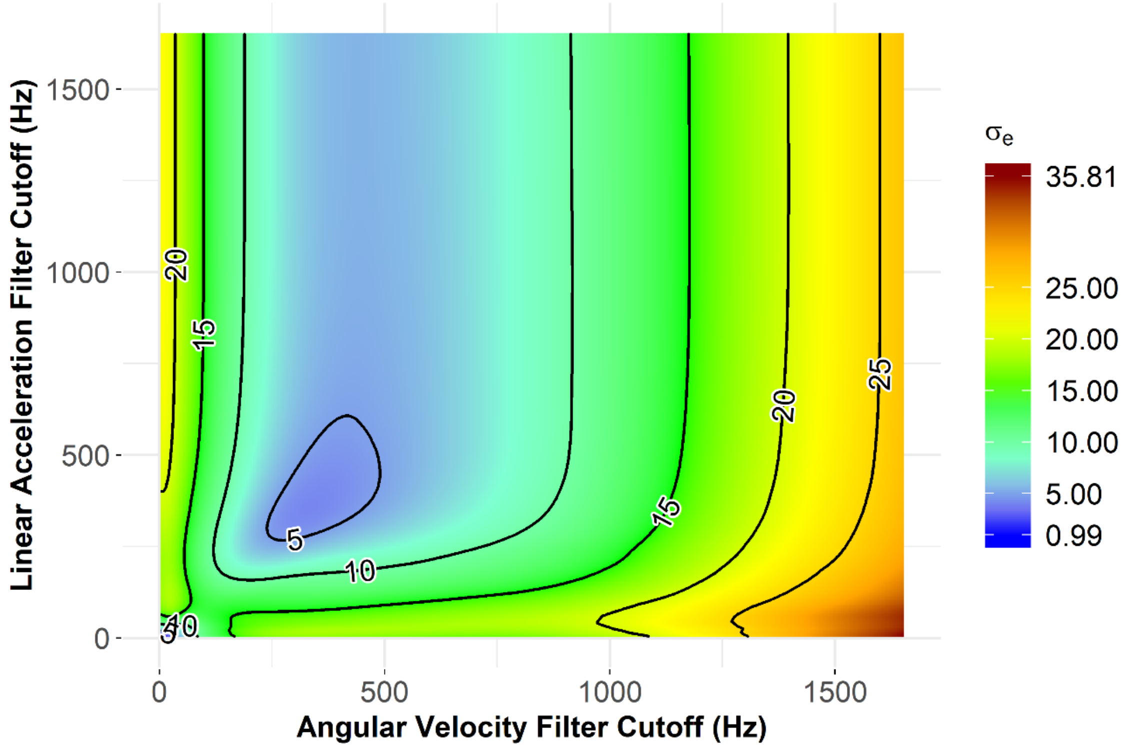

Fig. A2. Standard deviation of percent error for transformed peak linear acceleration by linear and angular cutoff frequency combinations. Contour lines represent 5% spacing; the colour bar runs from minimum to maximum standard deviation in percent error found in the dataset.

B. Filter Characteristics and Code

We constructed our filter to signal processing best practices, meeting SAE J211 digital filtering recommendations [12]. Though it did not explicitly use the pseudocode provided in the SAE J211 document, it did fall within the corridor for each specified CFC value from the document (Figure A3) [3]. The filter is a 4th-order lowpass, phaseless digital Butterworth filter. A phaseless effect was achieved through a double pass of the filter (once forward and once backward). The filter function corrects the cutoff frequency such that the double filter pass does not overly attenuate the output signal [12,13]. We have provided the code, adapted from [13], at the end of this document.

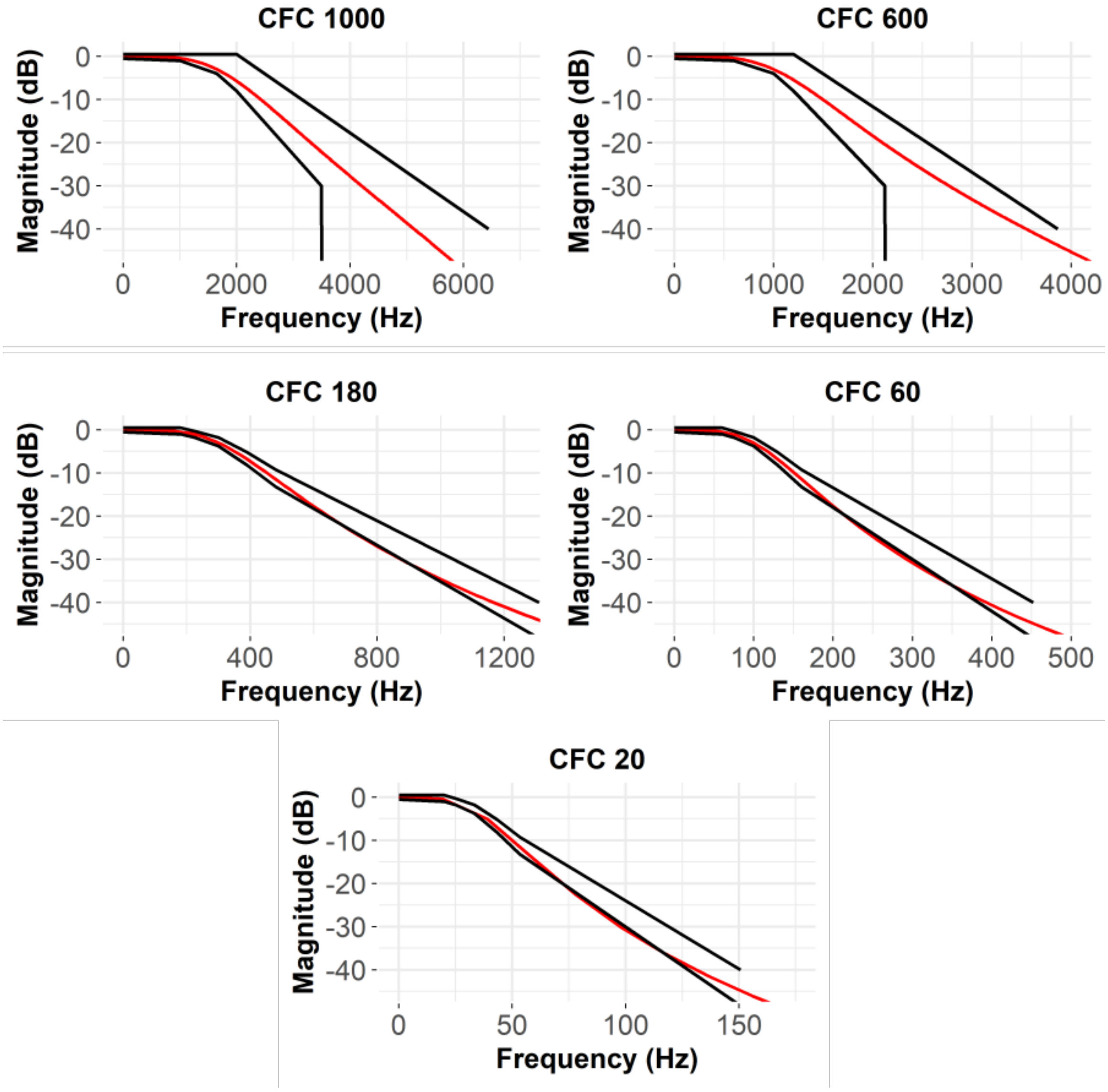

Fig. A3. The response of the 4th-order lowpass, phaseless digital Butterworth filter employed in this study falls within the corridors specified in SAE J211. The exception is for CFC values less than or equal to 180, where the filter response slightly over-attenuates higher frequencies. Cutoff frequency based on the -3dB point (F_H) was used to generate the equivalent filter for each CFC value in this plot.

```
function y = j211filter(x, fs, wc)
  % this function filters a signal using a 4th order lowpass phaseless
  % Butterworth filter consistent with SAE J211 specification
  % input:  x is the unfiltered signal
  %         fs is the sampling rate in Hz
  %         wc is the cutoff frequency in Hz (1.65*CFC)
  % output: y is the filtered signal

  % butterworth correction factor (David A. Winter, 2009)
  correction_factor = (2^(1/2))-1)^(1/(2*2);

  % corrected angular cutoff frequency
  Un = tan(pi*wc/fs)/correction_factor;

  % corrected cutoff frequency in Hz
  wc_corrected = atan(Un)*fs/pi;

  % compute filter coefficients and apply to signal
  [b,a] = butter(2,wc_corrected/(fs/2));
  y = filtfilt(b, a, x);
```