A New Approach to Restitution Coefficient Modelling in Low Intensity Collisions Between Vehicles

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Abstract The coefficient of restitution is an important parameter that must be estimated appropriately by a traffic accident reconstruction expert. Due to the limitations of data collection in this type of reconstruction, the development of appropriate models is required from easily obtainable parameters. We used data from impact tests of the Working Group on Accident Mechanics (AGU Zurich) in collaboration with the Test Centre (DTC) in Vauffelin/Bienne and the insurers Winterthur and Zurich. To our knowledge, these data have not yet been studied to relate the coefficient of restitution to new parameters such as the differences in mass between vehicles involved or the whether the impact has been partially or totally absorbed by elements designed for this task. In addition to the aforementioned factors, we found other parameters that influence the coefficient of restitution, such as the impact velocity (also a factor in previous works) and the age of the vehicles. In previous works, equations were used in order to obtain the coefficient of restitution, but recent advances in vehicle manufacturing techniques mean that these equations must be updated with new parameters that provide results that best fit the impact tests. We also compare previous similar studies on this topic.

Keywords Coefficient of restitution modelling, collinear collisions between vehicles, low intensity collisions, rear-end collisions.

I. INTRODUCTION

Low-intensity rear-end collisions are one of the most common causes of whiplash claims, and they have been studied in many papers on impact biomechanics for decades.

From a global point of view, the biomechanics of the impact can be divided into two main aspects: the medical part, which is based on how the human body is capable of receiving the energy transmitted to it during road traffic collisions and the production of injuries, and the technical engineering part, in which the transmission and absorption of energy in vehicles and their different components are measured to determine how much energy reaches the occupants.

The technical part of the collision can further be divided into two fundamental components: the impact velocity and the absorption and transmission of energies. The latter, specifically the so-called coefficient of restitution, is the focus of this work.

Although we deal with a specific type of collision, i.e., rear ends, and since there are different elements that can be damaged (and these have different technical characteristics), to determine the intensity of the impacts with the greatest possible accuracy, crashes must be studied according to the different elements that receive the energy of the collision.

Since the 1960s, studies have been carried out to determine the value of the coefficient of restitution as a function of impact velocity [1-8], but due to both improvements in vehicle construction techniques and the use of new materials, as well as the increase in quality and quantity of crash tests, the method of calculating the coefficient of restitution has had to be updated.

At the beginning of the 21st century, computer programmes for assessing the intensity of the impact between vehicles became popular, and even though these are very useful, they also require equations that model the crash mechanics. This paper presents a simple and easy-to-use method to determine one of the fundamental values in impact biomechanics: the coefficient of restitution.

II. THEORETICAL FRAMEWORK

In a particle system, when two of these impacts occur, there are three phases of collision. The first phase occurs prior to contact in which each particle has an initial velocity and a mass. The second phase is a period of

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deformation, in which the particles deform and have the same speed. Finally, there is a phase of restitution, in which, depending on the impact forces and the materials involved, the original shape is more or less recovered, and each impacted particle has a different final velocity.

Using the momentum equations of each particle and equating the initial and final momentum, we have:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{1}$$

where m_1 and m_2 represent the masses of the particles, u_1 and u_2 are their respective velocities before the impact and v_1 and v_2 are their respective velocities after it. Hereinafter, the subscript one will refer to the striking vehicle, while the subscript two will refer to the struck vehicle.

Therefore, we can use Equation (1) to obtain the expression of the coefficient of restitution, which depends on the velocities [9]:

$$e = \frac{v_2 - v_1}{u_1 - u_2} \tag{2}$$

The coefficient of restitution has a value between zero and one and is dependent on the materials of the impacting elements, the impact velocity, and the shape and size of the bodies, among other properties.

When the coefficient of restitution has a value of zero, the impact is said to be perfectly plastic, and consequently, the final velocities of the particles are the same, $v_1 = v_2$; this implies that there is no restitution period and that after the impact, the bodies stay together.

On the other hand, when the coefficient of restitution has a value of one, the impact is said to be perfectly elastic, and consequently, the velocity differences between the bodies before and after the impact are equal: $v_2 - v_1 = u_1 - u_2$.

Now, if instead of using the conservation of momentum, we study the equation of the kinetic energies before and after the collision, we have:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - Q \tag{3}$$

where Q is the loss of kinetic energy.

If we use the centre of mass as a reference system, we have the following equations:

$$v_{CM} = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} \tag{4}$$

$$u_{1,CM} = u_1 - v_{CM} = \frac{(u_1 - u_2)m_2}{m_1 + m_2} \tag{5}$$

$$u_{2,CM} = u_2 - v_{CM} = \frac{-(u_1 - u_2)m_1}{m_1 + m_2} \tag{6}$$

$$v_{1,CM} = v_1 - v_{CM} = \frac{-(u_1 - u_2)m_2e}{m_1 + m_2} \tag{7}$$

$$v_{2,CM} = v_2 - v_{CM} = \frac{(u_1 - u_2)m_1e}{m_1 + m_2} \tag{8}$$

where v_{CM} is the velocity of the centre of mass. We can now substitute Equations (5) and (6) into Equations (7) and (8), respectively, obtaining $v_{1,CM} = -eu_{1,CM}$ and $v_{2,CM} = -eu_{2,CM}$, so we can use the centre-of-mass version of Equation (3):

$$Q = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_2u_2^2$$

$$= \frac{1}{2} m_1 v_{1,CM}^2 + \frac{1}{2} m_2 v_{2,CM}^2 - \frac{1}{2} m_1 u_{1,CM}^2 - \frac{1}{2} m_2 u_{2,CM}^2$$

$$= \frac{1}{2} m_1 e^2 u_{1,CM}^2 + \frac{1}{2} m_2 e^2 u_{2,CM}^2 - \frac{1}{2} m_1 u_{1,CM}^2 - \frac{1}{2} m_2 u_{2,CM}^2$$

$$= \frac{1}{2} \left[m_1 \left(e^2 u_{1,CM}^2 - u_{1,CM}^2 \right) + m_2 \left(e^2 u_{2,CM}^2 - u_{2,CM}^2 \right) \right]$$

$$= \frac{1}{2} \left[m_1 u_{1,CM}^2 \left(e^2 - 1 \right) + m_2 u_{2,CM}^2 \left(e^2 - 1 \right) \right]$$

$$= \frac{1}{2} \left(e^2 - 1 \right) \left(m_1 u_{1,CM}^2 + m_2 u_{2,CM}^2 \right) \tag{9}$$

If we substitute Equations (5) and (6) into Equation (9), we obtain:

$$Q = \frac{1}{2} \left(e^2 - 1 \right) \left[m_1 \frac{(u_1 - u_2)^2 m_2^2}{(m_1 + m_2)^2} + m_2 \frac{(u_1 - u_2)^2 m_1^2}{(m_1 + m_2)^2} \right]$$

$$Q = -\frac{1}{2} \left(1 - e^2 \right) \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2 \tag{10}$$

This theoretical foundation is based largely on the conservation of momentum, that is, we assume that the system is closed and not affected by external forces and that the internal forces are not dissipative. This means that in a real impact, certain variations with respect to the theoretical results must be accounted for.

III. METHODS AND DATASET

To obtain the necessary parameters, collision tests were used between vehicles obtained from the database of the Working Group on Accident Mechanics (AGU Zurich), who worked with the Dynamic Test Centre (DTC) in Vauffelin/Bienne, and the insurers Axa Winterthur and Zurich in Switzerland [10].

These tests are clearly identified in the AGU Zurich database, and the most relevant information obtained from them is as follows:

- Collision type
- Brand and model of the vehicles
- Vehicle masses
- Impact velocity
- Vehicle speed changes
- Collision duration
- Coefficient of restitution
- Energy used in the deformation of materials

We accounted for crash tests in which the impact velocity is less than or equal to 21 km/h, the vehicles were in the same plane, and they were relatively new, using bumpers and materials closely related to current ones, meaning that the manufacturing dates of the vehicles must have been in 1998 or later. After applying this first filter, we obtained 97 crash tests. In all these tests, the struck vehicle, before the impact, was stationary, so $u_2 = 0$. Additionally, all the tests provided by AGU Zurich were fully aligned in the horizontal axis, and the angle of impact between vehicles was close to zero.

Once all the indicated crash tests were studied, they were divided into two different populations. One included collisions with aligned bumpers between both vehicles. The bumper is defined as the system composed of the bumper cover, the absorber and the bumper beam. The bumpers were aligned in both the vertical and horizontal axes. The other population included collisions with nonaligned bumpers (in at least one axis) and those impacts in which the struck vehicle was hit significantly below its bumper or those in which the main impact regions were different than the bumpers. Figure A-1 in the Appendix shows a sketch with a bumper-aligned collision (corresponding to the first population), while Figure A-2 and Figure A-3 show collisions with some offset in one of the axes (horizontal and vertical, respectively).

The first population is composed of 70 samples and the second one of 27 samples. See Table A-I in the Appendix for all the test names in each of the considered populations. We have limited our study to the first type of crash tests, and we have chosen those that follow the HS_XXX naming template (where XXX is a number) because crash tests complying with this naming template provide data results in a homogeneous way. This ultimately resulted in a total of 65 crash tests, comprising 23 brands and 60 different vehicle models.

To obtain the best possible result, several ways of determining different types of energies were compared, and the empirical kinetic energy loss is the result of applying the velocities and masses given in the crash tests to Equation (3). The theoretical kinetic energy loss is the result of applying the coefficient of restitution, the masses and the impact velocity to Equation (10). The value of the energy used in the deformation of the materials is obtained directly from the results table provided in the crash tests.

Finally, the energy fractions to which we will refer are the ratios of the previously mentioned energies to the initial kinetic energy.

The definition of the values of these energies will be used to estimate the coefficient of restitution and will be justified in the next section.

IV. RESULTS

The vast majority of previous studies [1][7][8][11] use the relation between the impact speed and coefficient of restitution to obtain an exponential formula similar to this one (we assume $u_2 = 0$):

$$e = \alpha \cdot \exp(-\beta \cdot u_1) \tag{11}$$

where α and β are dimensionless coefficients. However, in this study, we will show that we should account for additional factors to correctly determine the coefficient of restitution, such as the fact that the collision should be centred between the vehicle's bumpers. In cases where the impacts were off-centre or where the vehicle was struck below its rear bumper, we see a significant yet consistent decrease in the value of the coefficient of restitution, independent of other collision variables.

Different Populations Depending on the Alignment of the Bumpers

In the following figures, we visually represent the initial population of 97 samples. After the analysis of each of the impact tests to verify the use of aligned bumpers, it is suspected that the initial population should be divided into two different populations. In Table 1, we can see the main features for both populations.

As seen in Figures 1, 2 and 3, both populations seem different in terms of the value of their means, but we will justify this statement in a statistical way. Therefore, the objective of the following study will be to find statistical evidence that the means of the two populations are different with a high degree of probability. None of the outliers were removed in this study.

TABLE I

GENERAL CHARACTERISTICS OF THE POPULATIONS FOR THE COEFFICIENT OF RESTITUTION

	Number of samples	Mean	Standard deviation
Impacts without aligned bumpers	27	0.166	0.045
Impacts with aligned bumpers	70	0.311	0.067

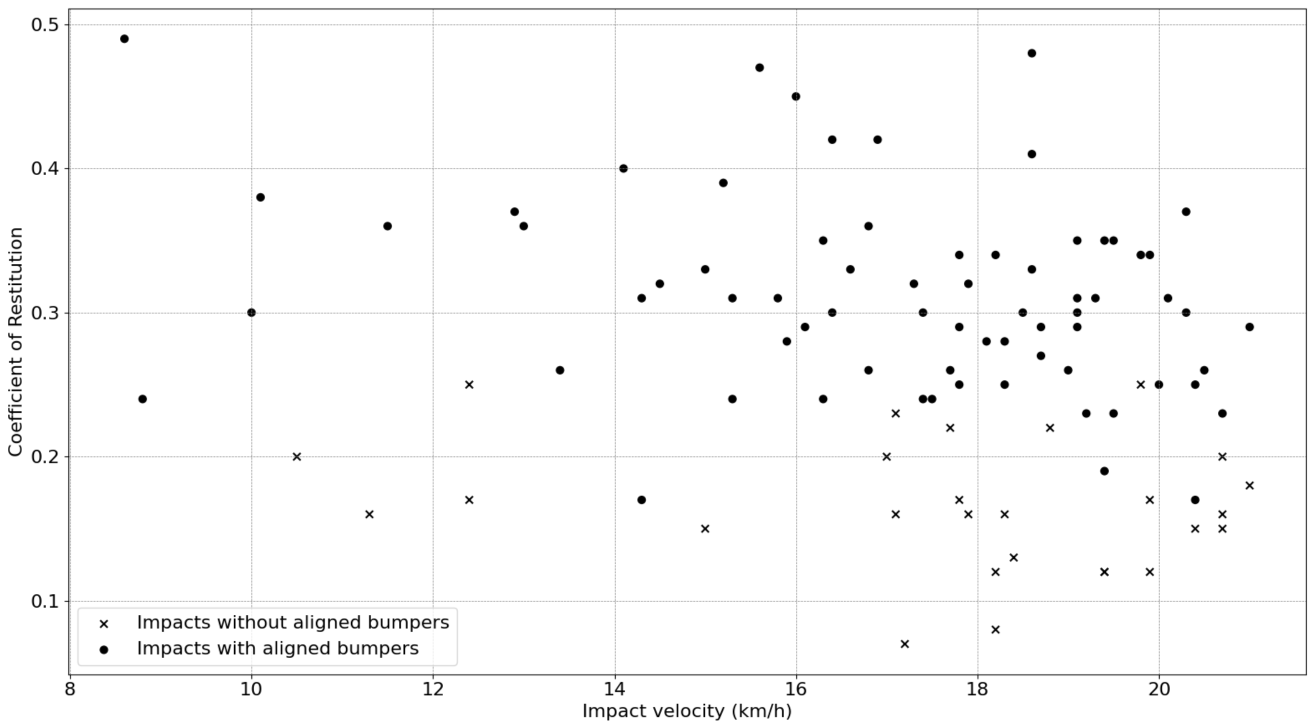

Fig. 1. The first crash test dataset split into two apparently different populations.

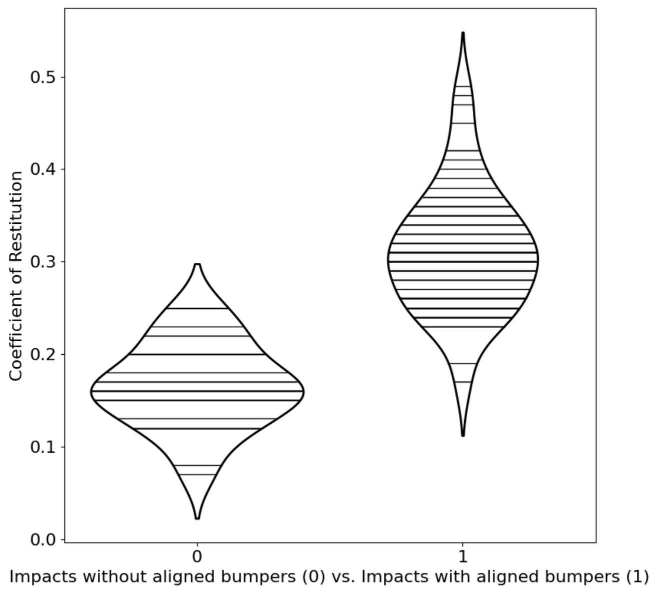

Fig. 2. Violin plots of both populations.

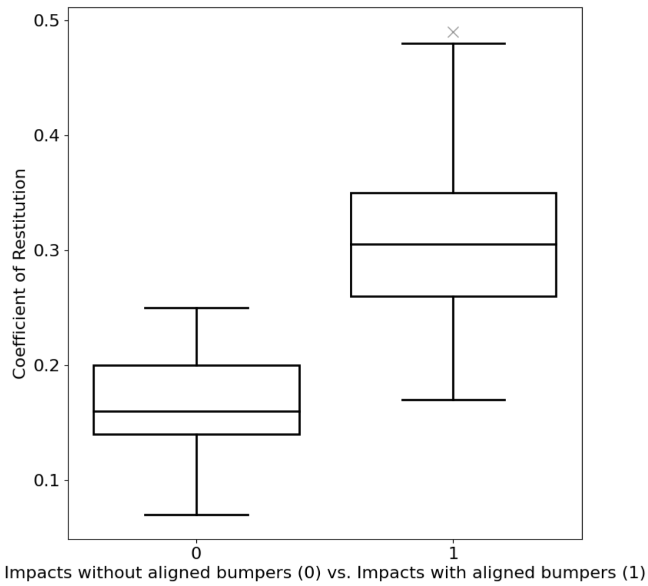

Fig. 3. Box-and-whisker plots of both populations.

The first step consists of testing the normality behaviour (i.e., whether the probability distribution follows a normal distribution) of both populations by running six tests on each. In all these tests, the null hypothesis is that the population follows a normal distribution with a level of significance $\alpha = 0.05$. The list of tests performed is [12-17]:

- Kurtosis
- Skewness
- Shapiro–Wilk test
- D'Agostino K–squared test

- Kolmogorov–Smirnov
- Anderson–Darling

TABLE II
P VALUES FOR NORMALITY TESTS

	Kurtosis	Skewness	Shapiro–Wilks	D'Agostino	Kolmogorov–Smirnov	Anderson–Darling
Impacts without aligned bumpers	0.996	0.989	0.520	1.000	0.726	0.707
Impacts with aligned bumpers	0.370	0.07*	0.09*	0.133	0.556	0.748

As seen (Table II), all the tests have been successful (p value greater than the significance level), although two p values (marked with an asterisk) are very close to α. Therefore, we believe that there is no statistical evidence to reject the null hypothesis: both populations tend to follow a normal distribution.

The next step is to check the homoscedasticity between the two populations, i.e., that the variances are equal. Now, the null hypothesis is that the variances of both populations are equal. The significance level α is, again, 0.05. We run four tests for homoscedasticity [18-21]:

- Levene
- Bartlett
- Fligner–Killeen
- Brown–Forsythe

TABLE III
P VALUES FOR HOMOSCEDASTICITY TESTS

	Levene	Bartlett	Fligner–Killeen	Brown–Forsythe
p values	0.065	0.027*	0.063	0.064

In this case, the p values in three of the four tests are slightly greater than or equal to 0.05; in addition, the p value of the Bartlett test is below α. Therefore, we believe that the results are not conclusive in favour or against the null hypothesis, so we continue with the study accounting for the two possibilities: that the two populations have the same variance and that the two populations do not have the same variance.

The last step is to check whether the two populations have the same mean (null hypothesis) or not (alternative hypothesis). The two statistical methods used assume that the populations follow a normal distribution (as we showed in the first step). Student's t test [22] assumes that the variances are equal, while the Welch variant [23] assumes that the variances are distinct. The significance level α remains the same at 0.05.

TABLE IV
P VALUES FOR TESTS ON THE MEAN

	t–Student	Welch
p values	4.5×10^{-17}	8.2×10^{-19}

We have obtained, in both tests, a p value that is much lower than α (almost zero). Therefore, we can say, as a conclusion, that we must reject the null hypothesis, and the two populations have different means and should not be evaluated together in the same study.

Estimating the Coefficient of Restitution

Once we justified that the initial population should be divided into two, we were able to continue our study with the 65 samples with the HS_XXX naming pattern in which the impact was also aligned from the point of view of the bumpers.

The objective was to obtain an exponential expression similar to Equation (11). The direct linear correlation between the impact velocity and coefficient of restitution gave us a very poor Pearson correlation coefficient: -0.2369. Therefore, we proposed an intermediate step between the impact velocity and the coefficient of restitution.

As we mentioned in the previous section, six variables related to energy were proposed to check which of them had the highest correlation with the coefficient of restitution:

- *Theoretical kinetic energy loss:* Obtained by substituting the impact test data into Equation (10).
- *Empirical kinetic energy loss:* Obtained by substituting the data from the impact tests in the energy balance, Equation (3).
- *Energy lost in deformations:* Obtained directly from crash test data.

The three respective fractions were also considered by dividing each energy by the kinetic energy before the impact. The fraction of empirical kinetic energy loss and the fraction of energy lost in deformations were calculated from impact test data, but the fraction of theoretical kinetic energy loss had to be developed mathematically.

We denoted F_t as the fraction of theoretical kinetic energy loss:

$$F_t = \frac{Q}{K_{initial}} = \frac{-\frac{1}{2}\left(1-e^2\right)\frac{m_1 m_2}{m_1+m_2}\left(u_1-u_2\right)^2}{\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2} \qquad (12)$$

where $K_{initial}$ is the kinetic energy before the impact. In all impact tests, the initial velocity of the struck vehicle is zero; therefore, $u_2 = 0$, so Equation (12) can be expressed in terms of only the coefficient of restitution and the masses of the two vehicles:

$$F_t = \frac{-\left(1-e^2\right)\,m_2}{m_1+m_2} \qquad (13)$$

For the sake of convenience, we studied this fraction in its absolute value in the following graphs:

$$\left|F_t\right| = \frac{\left(1-e^2\right)\,m_2}{m_1+m_2} \qquad (14)$$

Table V shows the Pearson correlation coefficients between each type of energy and the impact velocity u_1 (first column) and between each type of energy and the coefficient of restitution e (second column); these are direct linear correlations. The coefficients in the third column are calculated by multiple linear regression between u_1 and the respective energy as input variables to obtain e as the output variable.

TABLE V

PEARSON CORRELATION COEFFICIENTS (R)

	Impact velocity (u_1)	Coefficient of restitution (e)	Multiple linear regression
Theoretical kinetic energy loss	0.8779	-0.3637	0.4023
Empirical kinetic energy loss	0.8058	-0.2043	0.2379
Energy lost in deformations	0.8597	-0.3441	0.3629
Fraction of theoretical kinetic energy loss	0.7042	-0.6508	0.7216*
Fraction of empirical kinetic energy loss	0.6907	-0.2979	0.3010
Fraction of energy lost in deformations	0.5096	-0.4994	0.4998

The best correlation found in the table to obtain the coefficient of restitution is the theoretical energy loss fraction together with the impact velocity (marked with an asterisk).

We then calculated the linear regression using the fraction of theoretical kinetic energy loss as the output variable against the impact velocity as the input variable. Figure 4 shows this linear regression. We obtained the following equation (R = 0.7042):

$$\left|F_t\right| = 0.0180466077 \cdot u_1 + 0.1078234258 \tag{15}$$

where the velocity u_1 is expressed in km/h. We then used exponential regression to obtain the coefficient of restitution depending on the fractional loss of theoretical kinetic energy (R = 0.6345):

$$e = 0.9047853945 \cdot \exp(-2.5805866648 \cdot \left|F_t\right|) \tag{16}$$

Figure 5 shows this exponential regression. By substituting Equation (15) into (16), we obtained the following predictor:

$$e = 0.9047853945 \cdot \exp(-0.2782476948 - 0.04657083518 \cdot u_1) \tag{17}$$

Again, u_1 is expressed in km/h. We were then easily able to transform Equation (17) to fit the format given in Equation (11):

$$e = 0.6850214062 \cdot \exp(-0.04657083518 \cdot u_1) \tag{18}$$

If u_1 is given in m/s, we use the following equation:

$$e = 0.6850214062 \cdot \exp(-0.1676550066 \cdot u_1) \tag{19}$$

For the two results obtained (hypothesis tests for different means in populations and simple or multiple linear regressions), we used well-known statistical packages implemented in Python [24-25].

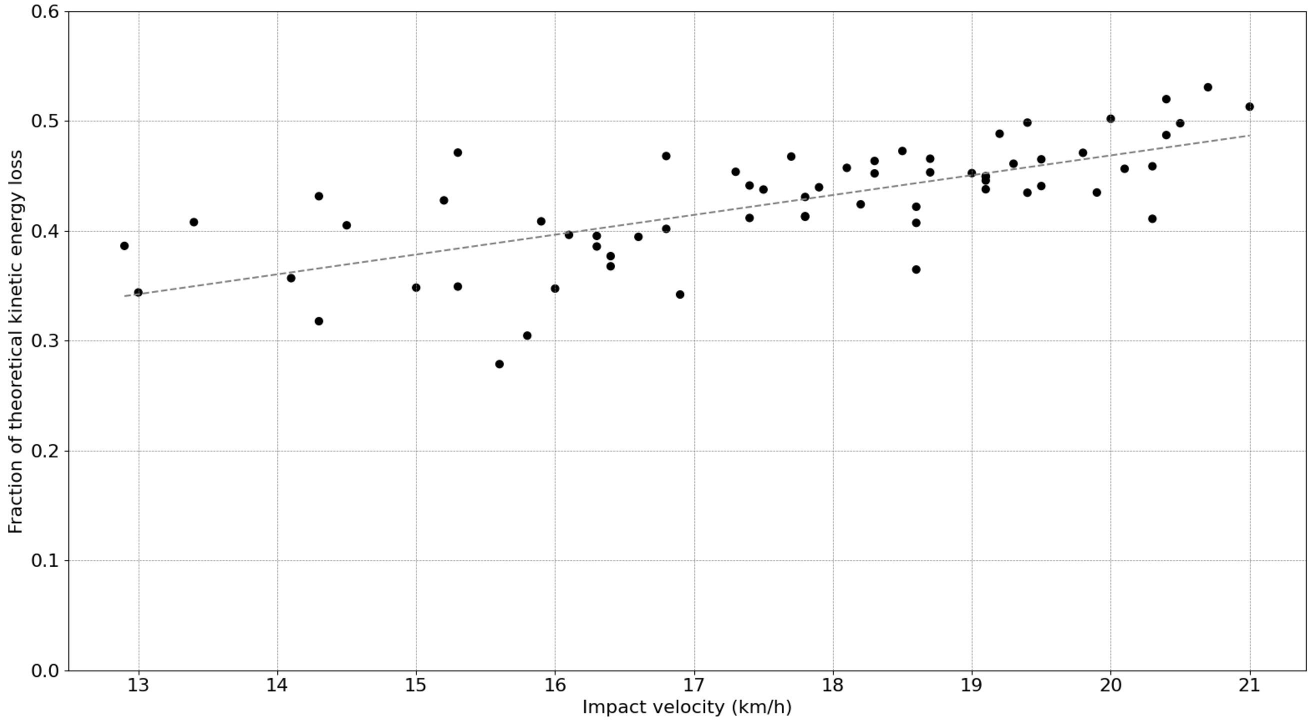

Fig. 4. Linear regression between impact velocity versus fraction of theoretical kinetic energy loss.

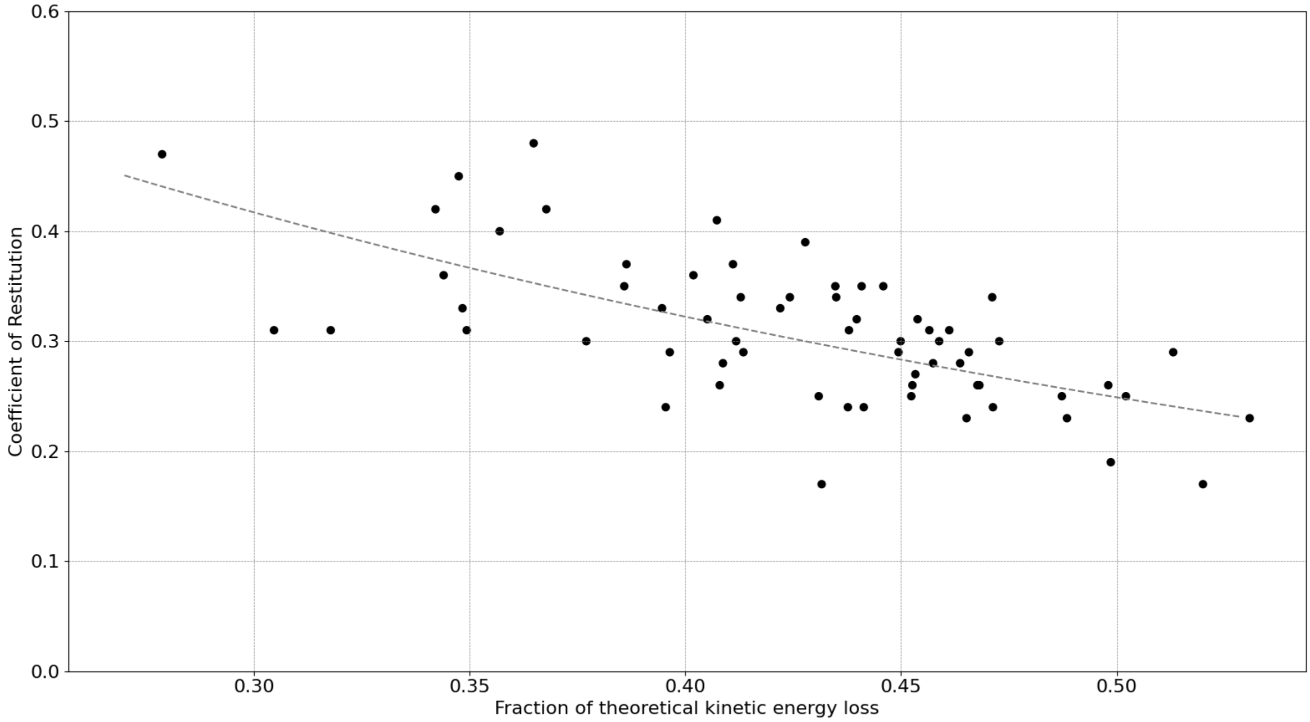

Fig. 5. Exponential regression for fraction of theoretical kinetic energy loss vs. coefficient of restitution.

V. DISCUSSION

Most of the studies prior to this one that estimate the coefficient of restitution did not have the opportunity to include the data from the most recent impact tests in their regressions or in the training of their predictors, but we are still able to compare the results of [1][7][8][11] with our results in the context of the state-of-the-art knowledge in this field. We perform a comparison based on the mean square error and the distributions of the mean and median. Our results use the predictor of Equation 17.

TABLE VI
COMPARISON WITH PREVIOUS WORKS ON THE ESTIMATION OF THE COEFFICIENT OF RESTITUTION

	Current study	[8]	[1]	[7]	[11]
Mean square error	0.0042	0.0113	0.0044	0.0085	0.0071
Prediction greater than the empirical data	31	60	39	57	12
Prediction less than the empirical data	34	4	26	8	53
Mean in prediction	0.3024	0.2220	0.2872	0.2408	0.3604

Our study has the best mean square error between the predictions and the empirical data (the measure of the coefficient of restitution in the impact tests), although [1] obtained a similar result. Examining the distribution on both sides of the median (predictions higher or lower than the empirical data), we find we have achieved a more balanced predictor by dividing almost 50% of the predictions on both sides of the curve described by Equation (17).

Furthermore, the mean of the coefficient of restitution of the empirical data is 0.3078; therefore, we managed to get closer to this value than the remaining studies (fourth row of the table).

Due to the lack of data that comes from real impact tests, the biggest limitation that we have found in our work is having to use the same data to train our predictor in the calculation of the regressions and to evaluate that predictor. It would therefore be desirable to have more data to be able to evaluate the predictor optimally or even to improve it in the training phase.

VI. CONCLUSIONS

We have justified in this study that a precise vertical and horizontal alignment of the bumpers of both vehicles is very relevant in terms of predicting the coefficient of restitution in low intensity impacts. The crash tests we used to develop the predictor include those of vehicles manufactured since 1998 to account for the fact that materials and manufacturing techniques have improved over time. In addition, the AGU Zurich impact tests used in this paper cover a wide variety of brands and models for both striking and struck vehicles.

The use of the theoretical kinetic energy loss fraction as an intermediate variable in regressions seems to predict and correlate empirical data in actual crash tests better than other studies in this field.

Following the same line of research, future work could include studies that account for bumpers that are not correctly aligned between the vehicles. In these cases, the impact energy is absorbed by bodywork elements that may not have been designed for this task, so the correlation coefficient should be predicted using other factors and variables.

An expression could also be sought for side impacts in which one of the bumpers is not involved in the crash so that the body parts and materials involved in the collision greatly differ from those considered in this work.

This study has been able to estimate the coefficient of restitution through easily obtainable parameters in an efficient way for current vehicles. The crash tests used are a good sample of what accident reconstruction experts encounter in their daily work. This estimate of the coefficient of restitution can be used to calculate the impact intensity, and the related impact biomechanics always depend on the impact velocity. Therefore, we need to also study how to estimate the impact velocity based on the damage observed after the impact.

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IX. Appendix

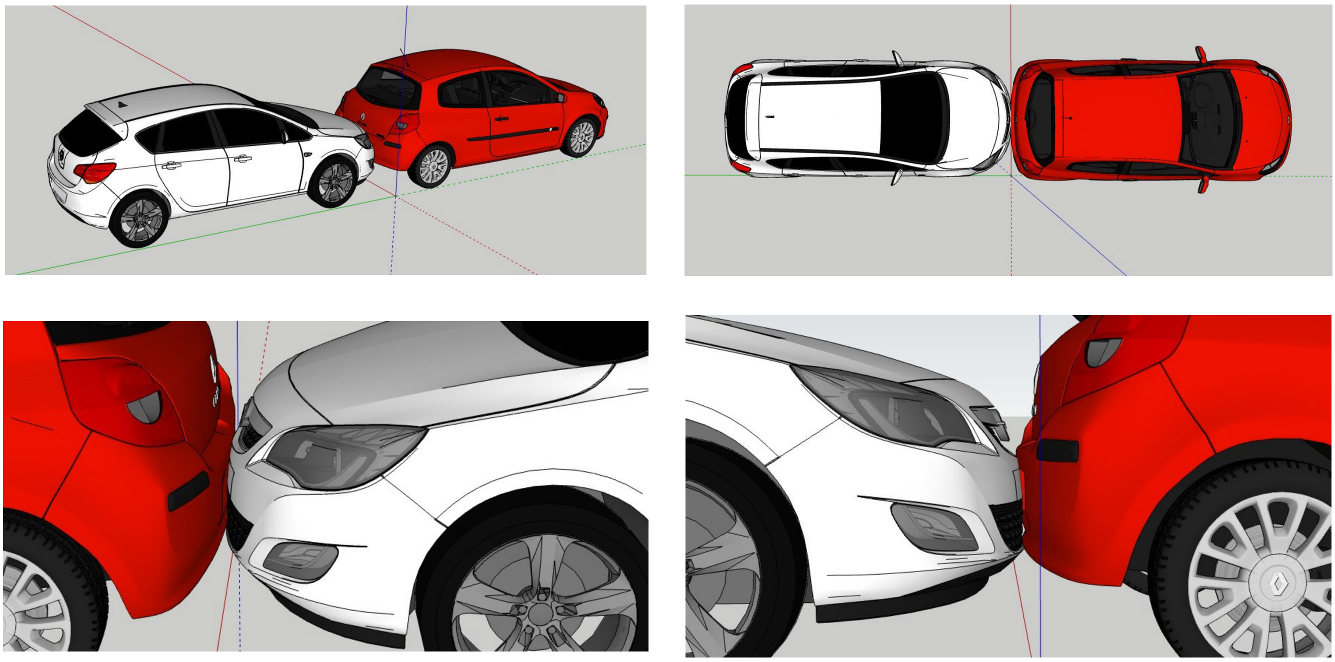

Fig. A-1. Sketch of a bumper-aligned collision.

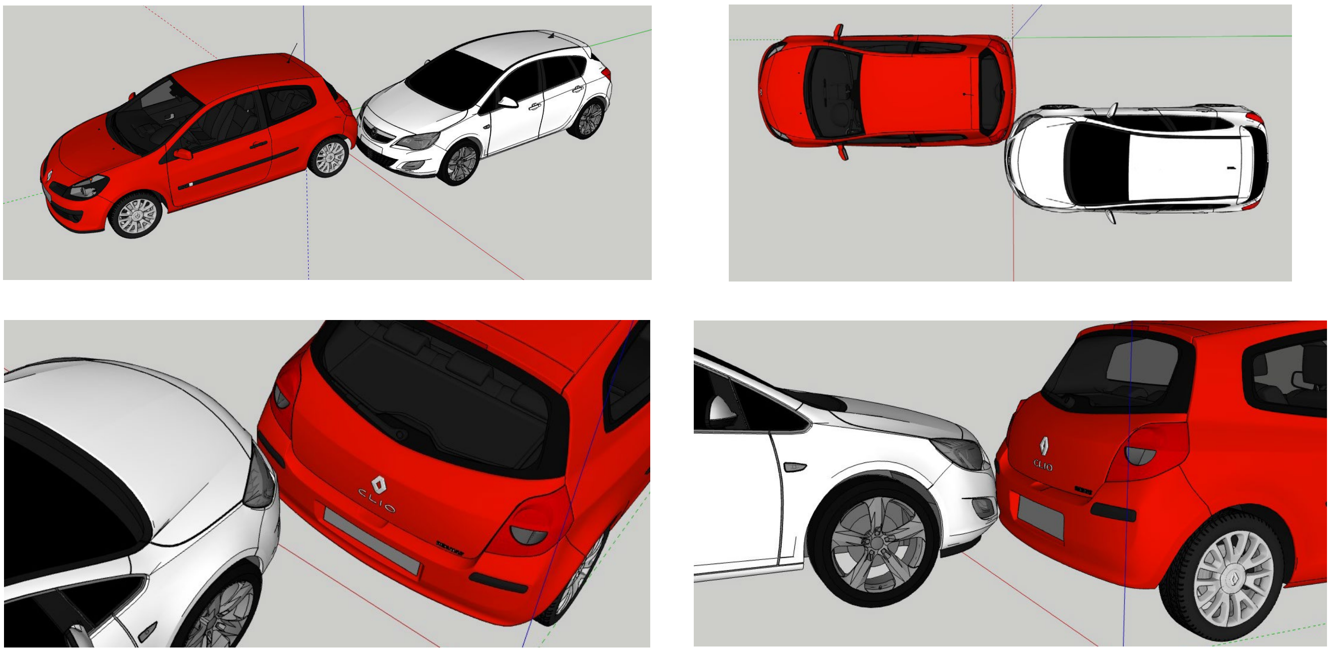

Fig. A-2. Sketch of a collision with bumpers not aligned along the horizontal axis.

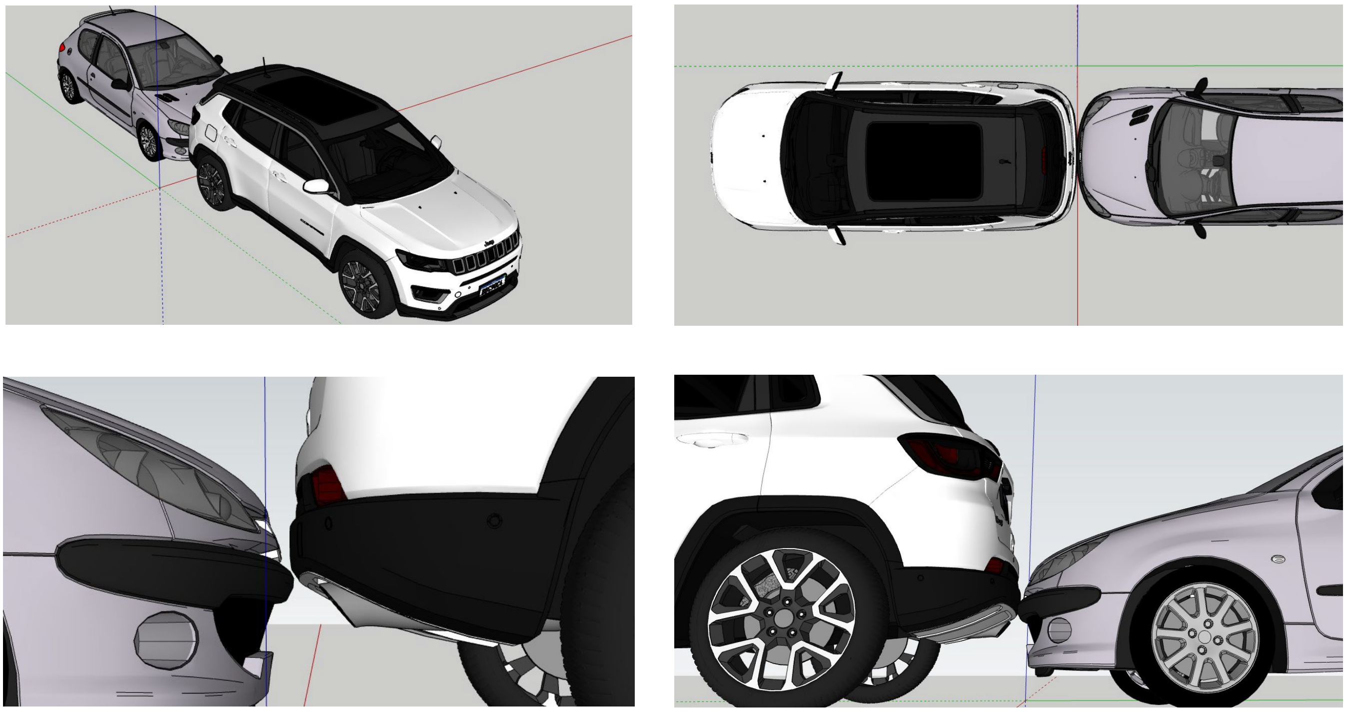

Fig. A-3. Sketch of a collision with bumpers not aligned along the vertical axis.

TABLE A-I
Crash Tests Used Depending on the Bumper Alignment

		Crash tests	
Impacts without aligned bumpers (27 crash tests)	AZT_04.15	HS_115	HS_155
	AZT_03.17	HS_125	HS_76
	AZT_03.19	HS_96	HS_87
	AZT_04.10	HS_83	HS_113
	AZT_04.14	HS_117	HS_111
	HS_91	HS_44	HS_109
	AZT_04.08	HS_119	HS_154
	HS_82	HS_55	HS_159
	HS_72	HS_59	HS_134
*Impacts with aligned bumpers (70 crash tests, those with * were finally discarded for the regression study due to different information templates)*	AZT_02.50*	HS_93	HS_153
	AZT_02.52*	HS_131	HS_58
	AZT_04.09*	HS_38	HS_147
	AZT_02.51*	HS_52	HS_112
	AZT_03.18*	HS_56	HS_151
	HS_22	HS_43	HS_54
	HS_36	HS_34	HS_114
	HS_DTC24	HS_160	HS_152
	HS_18	HS_95	HS_88
	HS_08	HS_77	HS_16
	HS_63	HS_146	HS_53
	HS_26	HS_133	HS_42
	HS_70	HS_128	HS_145
	HS_15	HS_103	HS_40
	HS_35	HS_66	HS_81
	HS_79	HS_110	HS_39
	HS_137	HS_65	HS_124
	HS_139	HS_89	HS_141
	HS_10	HS_30	HS_97
	HS_104	HS_148	HS_49
	HS_85	HS_143	HS_94
	HS_68	HS_123	HS_90
	HS_107	HS_102	HS_67
	HS_158		