A COMBINED EVALUATION METHOD AND A MODIFIED MAXIMUM LIKELIHOOD METHOD FOR INJURY RISK CURVES

Yuko Nakahira, Kazunori Furukawa, Hiroshi Niimi, Toshikazu Ishihara, Kazuo Miki
TOYOTA Central R&D Labs., Inc.

Fumio Matsuoka
TOYOTA Motor Corporation

ABSTRACT

A combined evaluation method and a modified Maximum Likelihood method were proposed for the sake of accurate estimation of the probability of injury risk curves in lower probability region, which are important for the improvement of the safety in car crash. According to the combined evaluation method, the modified Maximum Likelihood method had a tendency to yield better scores than the Maximum Likelihood method and the Mertz/Weber method.

Key Words : PROBABILITY OF INJURY, STATISTICS

IT IS IMPORTANT to obtain injury risk curves, the statistical relationship between the injury related parameter and the injury risk, in order to evaluate the performance in car crash safety. Two kinds of method have been utilized to calculate injury risk curves. One of them is the Maximum Likelihood method such as Logistic analysis, Probit analysis and Weibull analysis, and the other is the Mertz/Weber method. The Maximum Likelihood methods have been adopted to obtain injury risk curves since Versace (1971) applied the Logistic analysis (Lipson, 1973, pp.44-52) to the head injury data. Kroell (1986) utilized the analysis for the thoracic injury data and Cavanaugh (1990) for pelvic fracture data. The Probit analysis (Lipson, 1973, pp.22-32) was also applied to the clavicle fracture data by Lowne (1976). The Weibull analysis (Lipson, 1973, pp.36-44) was utilized to calculate injury risk curves for the head injury data etc. by Ran (1984). The Mertz/Weber method, on the other hand, was developed by Mertz and Weber (1982) to calculate injury risk curves. Mertz and Prasad (1997) applied the Mertz/Weber method to the neck injury data.

The standard to judge which method, the Maximum Likelihood method or the Mertz/Weber method, should be used has not been established. The characteristics of the two calculation methods are described below. The Maximum Likelihood methods provide high goodness of fit because the methods calculate the injury risk curves with the smallest error between the injury risk curves and the observed data. For that reason, the Maximum Likelihood methods have been utilized by many researchers. On occasion, however, the curves calculated by the Maximum Likelihood methods do not always approach zero when the injury related parameter approaches zero, e.g. the skull fracture risk curve (Hertz, 1993), the thoracic injury risk curve (Kroell, 1986). The occurrence of injury without any external force is contrary to a natural law of destruction.

Meanwhile, the Mertz/Weber method whose parameters of the cumulative normal distribution are determined by the median ranking values (Lipson, 1973, pp.17-18), always provides injury risk curves whose injury probability approaches zero when injury related parameter approaches zero. Nevertheless, the injury risk curves by the Mertz/Weber method, in principle, have less goodness of fit than the curves by the Maximum Likelihood method, the reason for using the median rank values is unobvious and the obtained risk curves are too sensitive to the two points selected from the injury

data (one is the maximum injury related parameter among the data in which injuries are not observed, the other is the minimum injury related parameter among the data in which injuries are observed).

As mentioned above, evaluation methods to regard the goodness of fit of the injury risk curves in lower probability region as important have not been discussed enough. Neither have calculation methods of the injury risk curves to estimate correctly in lower probability region. In this paper, a combined evaluation method to regard the goodness of fit of the injury risk curves in lower probability region as important was proposed and a modified Maximum Likelihood method as a calculation method of the injury risk curves to estimate correctly in lower probability region was developed. Applying the combined evaluation method to the injury risk curves calculated by the modified Maximum Likelihood method, the goodness of fit for the modified Maximum Likelihood method was estimated.

METHOD

ASSUMPTIONS FOR INJURY RISK CURVES :

As the nature of experimental injury data, it is obviously observed that the injury probability approaches zero when the injury related parameter approaches zero as well as that no injury occurs when injury related parameter is zero. We adopt the observed nature of injury data for injury risk curves as ASSUMPTION (A): "The injury probability approaches zero when the injury related parameter approaches zero."

It is also important that the difference between the injury data and obtained injury risk curves should be minimized. We also adopt it for injury risk curves as ASSUMPTION (B): "Obtained injury risk curves possess the maximum goodness of fit." The ASSUMPTION (A) is supposed to precede the ASSUMPTION (B) because we regard the lower probability region of injury risk curves as important.

COMBINED EVALUATION METHOD :

Based on the two ASSUMPTIONs introduced in the previous section, we develop an evaluation method for injury risk curves.

At first, we examine how to evaluate the ASSUMPTION (B). We introduce EB as an estimator of ASSUMPTION (B) and define the EB as equal to the log likelihood (Ran, 1984).

$$EB = \frac{1}{n} Log\left\{\prod_i Yi \times \prod_j (1 - Yj)\right\} \qquad (1)$$

Where Yi is a probability of injury at event i with injury, Yj is a probability of injury at event j with no injury, and n is the total number of the events. The larger the EB is, the higher the goodness of fit of the ASSUMPTION (B) is.

Secondary, we examine how to evaluate the ASSUMPTION (A). Suppose Y_0 is an estimated probability of injury at the time of injury related parameter zero. Given that all the experimental data accompany the evident event that injuries are not observed when injury related parameter is zero, the likelihood of Y_0 is expressed as

$$\prod_i (1 - Y_0) \times \prod_j (1 - Y_0) = (1 - Y_0)^n \qquad (2)$$

In order that the likelihood of Y_0 is valid with significant level α,

$$1 - \alpha \le (1 - Y_0)^n \qquad (3)$$

The inequality (3) is rewritten as

$$1 - (1 - Y_0)^n \le \alpha \qquad (4)$$

We define the left side of the inequality (4) as an estimator of the ASSUMPTION (A) and denote it as EA.

$$EA = 1 - (1 - Y_0)^n \qquad (5)$$

The inequality (4) and the equation (5) show that EA expresses the significant level of the closeness between Y_0 and zero. Provisionally, 5% is supposed as a significant level.

Because we regard the goodness of fit of the lower probability region of injury risk curves as important, the curves that satisfy , at first, the ASSUMPTION (A) with 5% significant level are selected, and then the goodness of fit of the selected curves is evaluated by the estimator EB. We will call the evaluation method described above as *Combined evaluation method*.

MODIFIED MAXIMUM LIKELIHOOD METHOD :

Based on the two ASSUMPTIONs introduced in the previous section, we develop a calculation method for injury risk curves.

Because the distribution of a population is important, it is desirable to presume a normal distribution that appears in nature universally. A normal distribution, however, does not satisfy ASSUMPTION (A) because the probability can not be zero when injury related parameter is zero. We examined the value of the probability regarded as substantial zero statistically at the time of injury related parameter zero.

Suppose the estimated injury probability is $P(0)$ when the injury related parameter is zero. The likelihood of $P(0)$ is expressed by replacing Y_0 with $P(0)$ in the equation (2). Similar to the inequality (3), the likelihood of $P(0)$ is

$$1 - \alpha \le (1 - P(0))^n \qquad (6)$$

The inequality (6) is solved about $P(0)$ as

$$P(0) \le 1 - (1 - \alpha)^{1/n} \qquad (7)$$

By substituting 0.05 for α in the inequality (5), a relation between n and $P(0)$ with the significant level 5% was calculated as shown in Fig. 1.

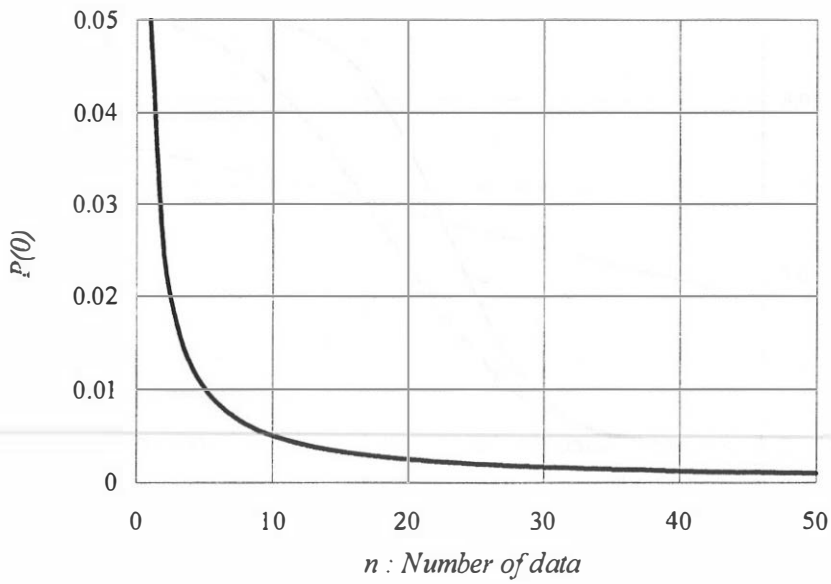

Fig. 1 - Relation between number of data and $P(0)$ (Probability on injury at the time of injury related parameter zero).

Applying the inequality (7), we improved the Maximum Likelihood method. In this paper, Logistic regression function is adopted as a Maximum Likelihood method. Because the function is easy to be arranged mathematically and provides similar regression curves with Probit regression function that presumes a normal distribution observed universally. Logistic regression function is expressed as

$$P(x) = exp(a + bx)/\{1 + exp(a + bx)\} \qquad (8)$$

Where x is an injury related parameter, a and b are coefficients and $P(x)$ is a probability of injury.

According to the ordinal Maximum Likelihood methods, the coefficient a and b are estimated so that the likelihood of the $P(x)$ achieves maximum. In this paper, the coefficient a can be calculated substituting zero for x in the inequality (8). The coefficient a is acquired as

$$a = ln\left(\frac{P(0)}{1 - P(0)}\right) \qquad (9)$$

The other coefficient b is calculated to satisfy the ASSUMPTION (B) of maximum likelihood. Consequently, the coefficients a and b can be determined. In this paper, the described method will be referred as *modified Maximum Likelihood method*.

RESULT

APPLICATION OF THE MODIFIED MAXIMUM LIKELIHOOD METHOD:

The three methods: the modified Maximum Likelihood method, the Mertz/Weber method and the Maximum Likelihood method were applied to the actual injury data which were used to evaluate the performance in car crash safety, and the obtained injury risk curves were compared. In this paper, Logistic analysis was adopted as a Maximum Likelihood method.

Head Injury Risk Curves: Injury risk curves of brain injury greater than or equal to AIS (Abbreviated Injure Scale) 3 vs. HIC (Head Injury Criterion) (Prasad, 1985), calculated by the three methods, were shown in Fig. 2. The modified Maximum Likelihood method and the Mertz/Weber method estimated probability of injury at less than 0.1%, although the Maximum Likelihood method estimated probability of injury at about 35%, when HIC was zero.

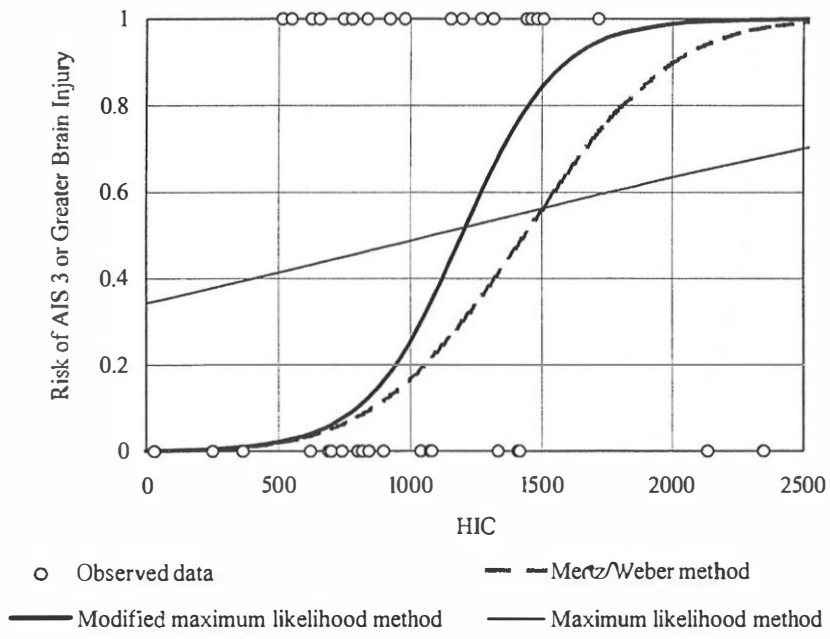

Fig. 2 - Injury risk curves of brain injury for HIC.

Neck Injury Risk Curves: Neck injury risk curves were calculated from Peak Neck Extension Moment (M_E) and N_{TE}(combined normalized neck tension and extension moment, N_{TE} = Peak Neck Tension/2120+M_E/26.8) data as shown in Fig.3 and Fig. 4. The data were experimented by Prasad and Mertz (1985), and improved by Vann (2000). The modified Maximum Likelihood method and the Mertz/Weber method estimated probability of injury at less than 0.2%, although the Maximum Likelihood method estimated probability of injury at about 2%, when N_{TE} was zero in Fig.3. Although the modified Maximum Likelihood method and the Mertz/Weber method estimated probability of injury at less than 0.1%, the Maximum Likelihood method estimated probability of injury at about 7%, when Peak Extension Moment was zero in Fig. 4.

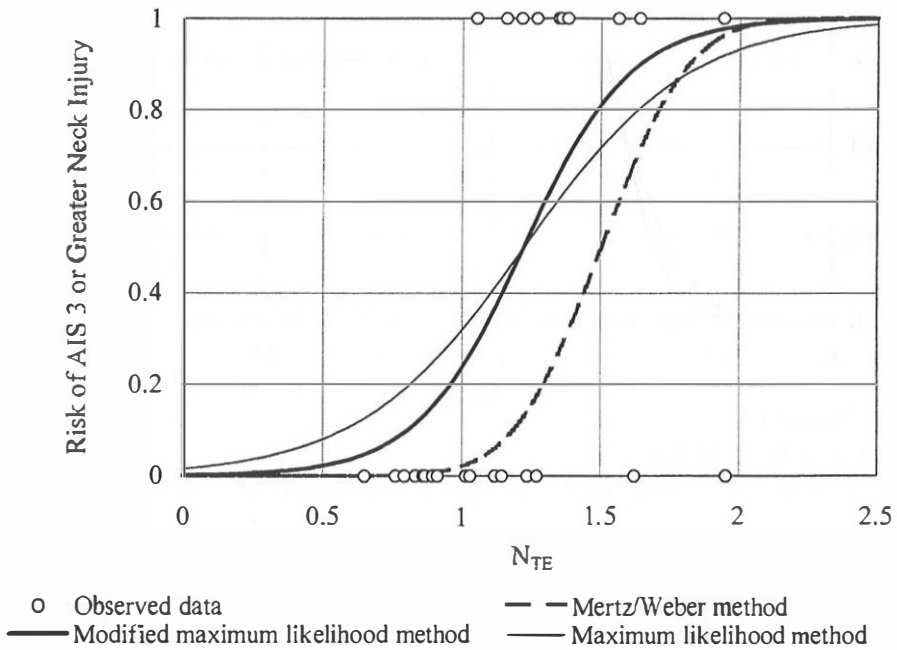

Fig. 3 - Injury risk curves of neck injury for N_{TE}

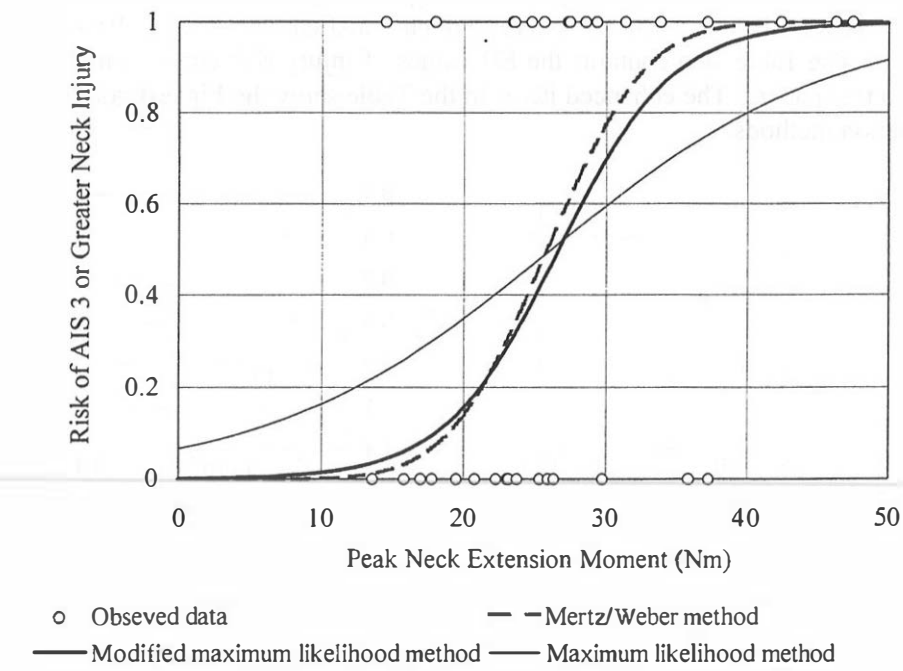

Fig. 4 - Injury risk curves of neck injury for Peak Neck Extension Moment

Injury risk curves of MAIS (the Maximum AIS) 4 or greater thoracic injury vs. VCmax (chest deformation velocity times chest compression) by Cavanaugh (1993), calculated by the three methods, were shown in Fig. 5. Although the modified Maximum Likelihood method and the Maximum Likelihood method estimated probability of injury at 0.01%, the Mertz/Weber method estimated probability of injury at about 0.0001%, when VCmax was zero.

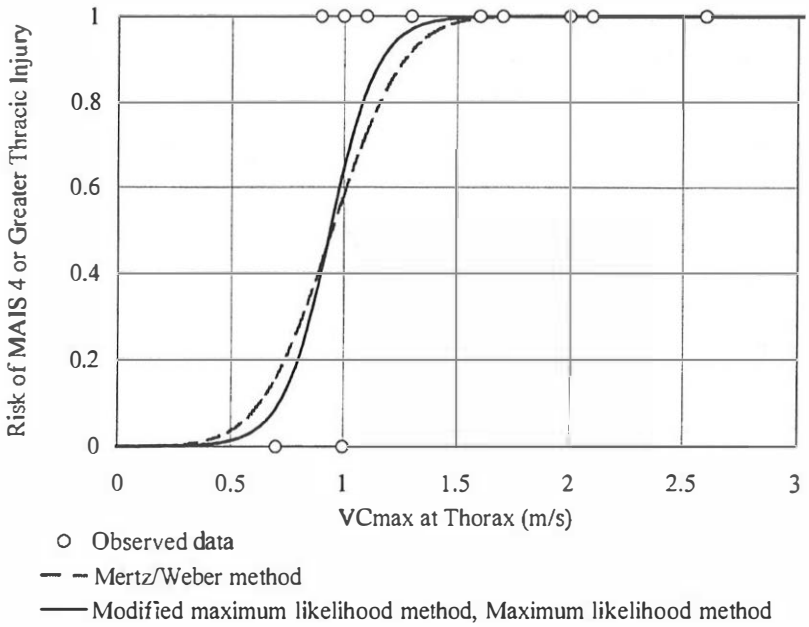

Fig. 5 - Injury risk curves of thoracic injury for VCmax

EVALUATION OF THE GOODNESS OF FIT:

The combined evaluation method was applied to the injury risk curves obtained by the three calculation methods: the modified Maximum Likelihood method, the Mertz/Weber method and the Maximum Likelihood method. The estimators of ASSUMPTION (A) and (B), EA and EB, were shown in Fig. 6 and 7, corresponding to Fig. 2 and 3 respectively.

The Table 1 shows EB values of the curves which satisfied the ASSUMPTION (A) with 5% significant level. The Table also contains the EB values of injury risk curves which were calculated but not shown in this paper. The enhanced items in the Table show the highest goodness of fit among the three calculation methods.

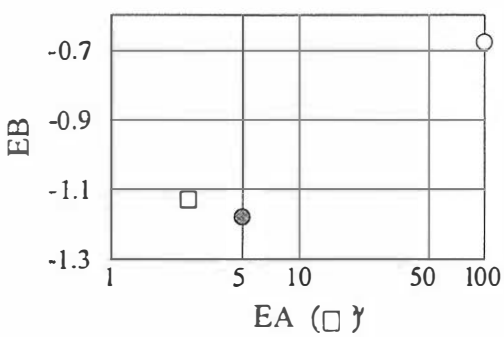

Fig. 6 - The estimators of injury risk curves in Fig. 2

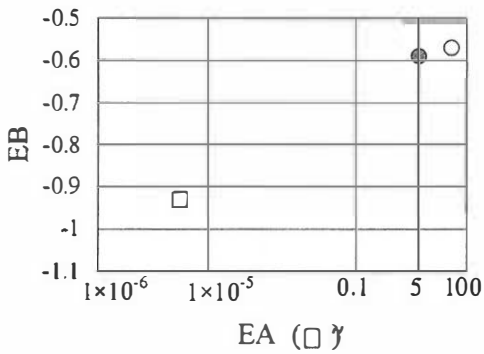

Fig. 7 - The estimators of injury risk curves in Fig. 3

Table 1 - The EB values of the injury risk curves satisfied ASSUMPTION (A).

Boby part	Risk Curve		Maximum Likelihood method	Modified Maximum Likelihood method	Mertz/Weber method
Head	HIC vs. Brain Injury	(Fig. 2, 6)	-	-1.177	-1.131
	HIC vs. Skull Fracture	(Prasad, 1985)	-	-0.832	-0.760
Neck	N_{TE} vs. Neck Injury	(Fig. 3, 7)	-	-0.590	-0.931
	Ext. Moment vs. Neck Injury	(Fig. 4)	-	-0.709	-0.788
	Tension vs. Neck Injury	(Vann, 2000)	-0.269	-0.269	-0.353
	KF vs. Neck Injury	(Vann, 2000)	-	-0.531	-0.825
Thoracic part	VCmax vs. Thoracic Injury	(Fig. 5)	-0.244	-0.244	-0.253
	TTI vs. Thoracic Injury	(Cavanaugh, 1993)	-0.207	-0.207	-0.300
Foot	Axial Force vs. Foot-ankle injury	(Yoganandan, 1996)	-	-0.506	-0.564

: The highest EB value among the three methods.

Confidence limits: The confidence limits of the injury risk curves by the modified Maximum Likelihood method can be calculated by the usual calculation method of confidence limits (Lipson, 1973, pp.387-391) as shown in Fig. 8.

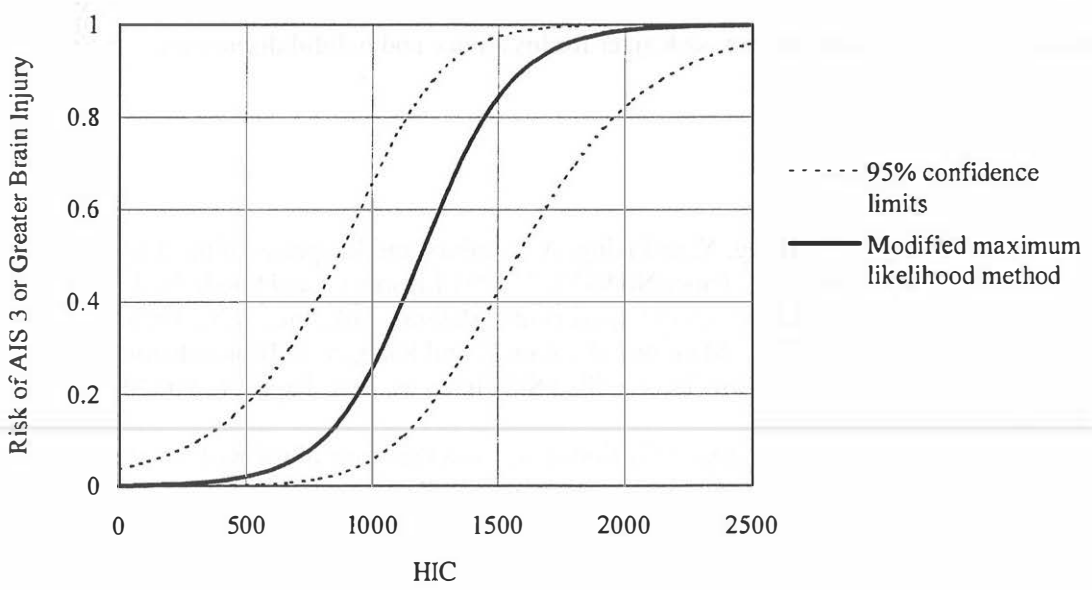

Fig. 8 - Confidence of limit for the risk curve of brain injury
(by modified maximum likelihood method in Fig. 2)

DISCUSSION

The combined evaluation method employs both of the estimators of the ASSUMPTION (A) and (B). The EA values of the curves calculated by the modified Maximum Likelihood method were less than or equal to 5% in Table 1. On the contrary, the EA values of the curves calculated by the Maximum Likelihood method were not always less than or equal to 5%. It is natural that the calculated curves by the modified Maximum Likelihood method satisfy the ASSUMPTION (A) because the method can attain any significant level.

With regard to the EB values, the modified Maximum Likelihood method offered the highest EB values in most injury cases in Table 1. Although the two exceptions in which EB of the curves calculated by the modified Maximum Likelihood method were not maximum, the EB values of the modified Maximum Likelihood method are as high as those of the Mertz/Weber method. The modified Maximum Likelihood method, based on the methodology itself, provides the injury risk curves with higher likelihood.

According to the combined evaluation method which employed the estimators, EA and EB, the modified Maximum Likelihood method had a tendency to yield better scores than the Maximum Likelihood method and the Mertz/Weber method.

The modified Maximum Likelihood method is considered to have both the advantage of the Maximum Likelihood method that the goodness of fit is high and the advantage of the Mertz/Weber method that the probability of injury approaches zero when injury related parameter is zero.

CONCLUSION

In this paper, in order to estimate correctly the probability of injury in lower probability region of injury risk curves, based on the nature of experimental injury data, the combined evaluation method was proposed to estimate the goodness of fit in lower probability region of the injury risk curves, and the modified Maximum Likelihood method as a calculation method for injury risk curves was developed. According to the combined evaluation method, the modified Maximum Likelihood method had a tendency to yield better scores than the Maximum Likelihood method and the Mertz/Weber method.

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