CAR SIZE and RELATIVE SAFETY:
- FUNDAMENTAL THEORY and REAL LIFE EXPERIENCE COMPARED

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ABSTRACT

In this paper the passive safety of cars is examined in the context of the influence of car size and mass on the relative safety of cars. The fundamental relationships of Newtonian mechanics are used to derive a generalised equation for the relative safety of cars of different sizes when involved in frontal collisions. Further equations are derived for collisions between cars of similar size and for single vehicle crashes. These are combined with overall injury criteria to give a series of predicted Relative Injury Risk relationships. Theory shows that in collisions between cars of similar size and in single vehicle accidents the fundamental parameter which determines Relative Injury Risk is the size, i.e. the Length of the car whereas in collisions between dissimilar sized cars the fundamental parameters are the Masses and the Structural Energy Absorption properties of the cars. The paper postulates that there are two different phenomena for the relative energy absorption of the cars, the first based on the dominance of the crushing forces imposed on the structures and the second based on the dominance of the inertia forces generated by the collapsing front structures.

The predictions from the theoretical models are compared with the results of field evaluations of Relative Injury Risk to car occupants carried out in the U.S. and in Europe for car to car and single vehicle collisions. There is a high level of correlation between the theory and the field evaluations of Relative Injury Risk. An explanation is provided for the form of the probability distribution for injury severity reported by Evans (1994,a) and is shown to provide correlation between the crash severity/injury severity characteristics of the U.K. and U.S. car collision populations.

DESpite progress in accident prevention it is inevitable that collisions will take place and that injury will result. As the preponderance of injury producing accidents to car occupants are frontal collisions this paper examines the influence of car size and structural crush behaviour on the safety of car occupants in frontal collisions. The paper extends previous analyses (Wood 1993 c,1995 a,b) by considering both the dynamic crush and the inertia effects of the crushed portions of the car on the car to car interaction.

INJURY CRITERION

The biomechanical injury causing mechanisms differ for the various body regions and organs. However as we are concerned with the overall passive safety of cars it is appropriate to use an overall measure of injury severity.

The forces exerted on the person's body can be normalised in terms of the accelerations imposed on the body. These accelerations vary over the course of the collision. Research has shown that injury severity is related to acceleration level and, for example, acceleration levels of 80g for 3 ms. are representative of the boundary conditions for head injury for the majority of the population. Other work originated by Gadd (1966) has shown that injury severity is related to the 2.5 power of average acceleration times the impact duration.

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However as the time durations of the preponderance of car collisions are very similar the overall injury severity can in the first instance, be regarded, (based on research by Gadd, 1966), as being proportional to the average acceleration imposed on the person's body to the power of 2.5. In general terms therefore an overall injury criterion can be considered as,

\[ \text{Injury Severity} = (\text{Average Body Acceleration})^{2.5} \quad (1) \]

Considering impacts where the restrained occupant does not contact the car interior the forces and accelerations applied to the occupant are nominally related to the total distance moved by the occupant during the collision. This is the sum of the amount of crumpling of the car front and of the forward motion of the occupant within the interior of the car. However research has shown (Wood 1993a) for occupants restrained by seat belts that the average acceleration imposed on the occupant is related to the average acceleration of the car structure during crumpling. In this regard the greater the crushing of the car front at a specific impact severity the lower the acceleration imposed on the car structure and hence on the restrained occupants.

ACCELERATION EQUATIONS

The previous section showed that in general the injury severity of the car occupants can be regarded as being related to some function of the average acceleration imposed on the car occupant compartment during impact. Newtonian mechanics shows that the average acceleration imposed on a car body is,

\[ \bar{a}_b = \frac{E_b}{\bar{M}_b \cdot d_b} \quad (2) \]

where \( \bar{M}_b \) is the mean mass of the car body, \( E_b \) is the energy absorbed by the body and \( d_b \) the displacement of the body. Because cars crumple during impact and these crumpled portions have mass which experience extremely high accelerations once they start to crumple, the forces imposed on the car front are different and higher than those applied to the uncrushed portions of the car. It is these latter forces and the movement of the uncrushed portions of the car which determine the energy absorbed, \( E_b \), in decelerating the uncrushed occupant compartment. The other element of energy absorption is the deceleration of the crushed elements of the structure. Consequently \( \bar{M}_b \) is the mean mass of the uncrushed parts of the car, not the total mass. The displacement, \( d_b \), is the dynamic displacement of the occupant compartment. The permanent or residual crush which can be measured on the car after the collision is often used as a surrogate for dynamic displacement but is always less than the dynamic displacement.

Considering a pair of cars colliding together, a case car, \( c \), and a partner car, \( p \). The ratio of accelerations is,

\[ \frac{\bar{a}_c}{\bar{a}_p} = \left( \frac{\bar{M}_p}{\bar{M}_c} \right) \cdot \left( \frac{d_p}{d_c} \right) \cdot \left( \frac{E_c}{E_p} \right) \quad (3) \]

where for the reasons detailed above
This relation shows that the acceleration ratio between the two cars and hence the relative injury risk is a function of the ratio of the average masses of the uncrushed portions of the two cars involved and of their energy absorbing properties. In other words the fundamental parameters which influence relative injury severity are the mass ratio of the two cars and the energy absorbing properties of the interacting pair.

Turning now to the special cases of collisions between cars of equal size and mass and of single car crashes into solid objects we can obtain the acceleration ratio for crashes of cars of different sizes involved in single vehicle accidents and for crashes between different sized pairs of equal size cars. This, for similar mass and structural effects, is,

\[
\frac{a_c}{a_p} = \left(\frac{v_c}{v_p}\right)^2 \cdot \left(\frac{d_p}{d_c}\right)
\]

Here the relative acceleration ratio and hence relative injury risk is related to the collision speeds and to the deformation of the cars. The masses of the cars are not involved. If we consider for the population of accidents that the distribution of collision speeds is independent of car size then equation 6 takes the form,

\[
\frac{a_c}{a_p} = \frac{d_p}{d_c}
\]

where \(d_c\) and \(d_p\) are the dynamic crush displacements of the respective cars or pairs of cars.

**CHARACTERISTICS of the CAR POPULATION**

**DIMENSIONAL AND MASS CORRELATIONS** The dimensional, mass and crushing characteristics of the car population and of individual car types are important parameters. Kahane (1991) and other researchers have shown that there is direct correlation between the wheel base and the wheel track dimensions of cars of different sizes and that both correlate with the curb mass of the cars. The crushing behaviour of cars in frontal collisions is influenced by the design of the front structures of the car and the distances from the front of the car to the front longitudinal struts, the engine, the front bulkhead/firewall and leading edge of the door. Wood (1993,b) has shown that, for the car population, these distances are proportional to car length with mean values of 4.7%, 11.4%, 32.0% and 38.0% respectively. Wood (1993,c) has shown that the curb mass of the car population is proportional to the overall length to the power of 2.48 while Evans (1994,b) shows that mass is proportional to wheelbase to the power of 2.51 and which is 60% of overall length. These studies show that the overall length of the car population can be considered to be, on average, proportional to the curb mass to the power of 0.4, i.e.,

\[
L \propto \text{Mass}^{0.4}
\]

**MASS VARIATION** Many studies of crashworthiness implicitly assume that the car consists of an occupant compartment of mass equal to the mass of the car with a massless crushable structure attached to the front. Evaluation of the yaw inertia properties of the car population by Wood (1992,a), shows that cars can be regarded as having uniformly distributed mass. As
the car crumples under impact the mass of the structure remaining to be decelerated decreases as the crushing progresses. This has been confirmed by Bismuth (1994) and by Fossat (1994) who show that the effective mass of the car reduces as the crushing progresses. This also highlights that the inertia forces required to decelerate the crumpled portions of the car should be taken into account in the force balance between the opposing car fronts. Wood (1993,c) has shown for a uniformly distributed mass car the ratio of the mean uncrushed mass to the original mass of the structure is,

\[ \frac{M}{M_0} = \frac{d}{L} \ln\left( \frac{L}{L-d} \right) \] (8)

The implications of this relation are, for example, when the dynamic displacement of the car is 20% of its overall length that the acceleration of the occupant compartment at maximum dynamic crush is 25% higher than where the mass remains constant.

ENERGY ABSORPTION PROPERTIES Examination of the energy absorbing properties of the car population, the relationship between speed and normalised crush depth \(d/L\),

\[ V_{oes} = 4.6 + 119.1 \left( \frac{d_{residual}}{L} \right)^{\frac{2}{3}} \] (9)

has been shown, Wood (1992,a,b), to describe the overall barrier crush behaviour of the car population. It has also been shown that this relation can be used for the normal range of crush profiles obtained in frontal crashes to estimate collision speeds (Wood 1992,b). Separately Moore (1970) has also shown that there is a \(2/3\) power relation between impact speed and crush depth. Equation 9 shows that the average structural characteristics of the car population are independent of car size, Wood (1995b) albeit that the behaviour of any individual car type will differ from this average characteristic.

RESIDUAL AND DYNAMIC CRUSH CORRELATION From the viewpoint of car size and safety, equation 9 shows that, on average, the residual crush in a collision of given severity, is proportional to the overall length of the car in question. However from the viewpoint of the accelerations imposed on the occupant compartment we are concerned with the dynamic crush. Wood (1992,a) has shown that a strong correlation exists between dynamic and residual crush and this can be described as,

\[ \frac{d_{dynamic}}{L} = 0.03 + 1.06 \left( \frac{d_{residual}}{L} \right) \] (10)

In determining the accelerations imposed on the car structure it is the dynamic crush which is the determining crush level.

THEORETICAL RELATIVE INJURY RISK EQUATIONS

By combining the fundamental theory and the characteristics of the present car population with the injury criterion based on Gadd (1966) we can arrive at theoretical relations for relative injury severity/risk and for the manner in which it varies with size and speed.
SIMILAR CAR TO CAR AND SINGLE CAR COLLISIONS

Because of equation 9 for a given collision speed the depth of the car crush is proportional to overall length. Therefore for single car crashes into rigid objects or for collisions between pairs of similar sized cars equation 6 takes the form,

\[
\frac{a_c}{a_p} = \frac{L_p}{L_c}
\]

(11)

Combining this with Injury Severity as being to the 2.5 power of acceleration gives,

\[
\frac{\text{Injury Severity case car}}{\text{Injury Severity partner}} = \text{Relative Injury Risk} = \left(\frac{L_p}{L_c}\right)^{2.5}
\]

(12)

The theory indicates that this phenomenon is fundamentally a size or length effect and is not a mass effect. However because of the very strong correlation between mass and length, mass terms can be used to replace length in the equation and give,

\[
\text{Relative Injury Risk} = \left(\frac{M_p}{M_c}\right)
\]

(13)

This equation shows that the relative injury risk in single car crashes into rigid objects and in collisions between pairs of similar sized cars is inversely related to the mass of the cars where mass is, in this instance, a surrogate for length.

COLLISION SPEED AND INJURY RISK

Considering impact severity in terms of collision speed, equation 5 shows that average acceleration is proportional to \( V^2 \) divided by the dynamic crush displacement. In equations 9 and 10 we have relations for the energy equivalent speed \( V_{ee} \) and normalised residual crush \( d/L \) and between dynamic and residual crush while equation 8 accounts for the reduction of the mass of the uncrushed portions of the car with crush depth. Wood (1993, b) and O’Riordain (1994) have shown that the differing shapes of the front of the deforming car in various frontal collisions, full width, narrow object, etc. can be represented by simple geometric shapes and that the crushing force-deformation characteristics thus calculated closely match the actual force characteristics. Substituting these equations for the two extreme crush shapes, flat full width and angled crush profile gives the extreme relationships between acceleration and \( V_{ee} \). These are,

\[
\bar{a} \propto \left(\frac{1}{L}\right) \cdot (V_{ee})^{0.73/0.94}
\]

(14)

Equation 14 shows that acceleration is proportional to \( V_{ee} \), the energy equivalent speed and hence to \( \Delta V \) and to the collision closing speed to the power of 0.73 for the flat crush profile and 0.94 for the angled profile, an average of 0.83, and is inversely related to the overall length of the car. When combined with the 2.5 power relation between acceleration and injury severity we obtain the injury severity relation,

\[
\text{Injury Severity} \propto \left(\frac{1}{M_c}\right) \cdot (V_{ee})^{1.83/2.34}
\]

(15)

Equation 15 indicates that injury severity increases with energy equivalent velocity, \( V_{ee} \) and \( \Delta V \) to the power of 1.83/2.34, an average of 2.08, and with the inverse of the mass of the car.
This relation shows that there are an infinite family of injury severity-velocity relationships for the range of masses found in the car population, refer to Figure 1.

Returning to equation 15 manipulation shows for any specific car or car population taking the ratio of injury severity to maximum severity (fatality) gives the equation,

$$Relative \ Injurti \ Severity = \left( \frac{v_{ees \ max}}{v_{ees \ max}} \right)^{1.83/2.34} \ (16)$$

where $v_{ees \ max}$ is the mean speed for fatality (probability = 0.5) in the car group or population being examined and is proportional to mass to the power of $1/n$ where $n = 1.83/2.34$.

**Figure 1. Theoretical Family of Injury Severity - Velocity Relationships**

**COLLISION SPEED AND INJURY PROBABILITY** Equation 15 also indicates that the overall equations for the probability of either serious or greater injury (AIS 3+) or of fatality as a function of $\Delta V$ reported by Evans (1994,b) and by Joksch (1993),

$$P(i) = \left( \frac{\Delta V}{\alpha} \right)^{k} \ (17)$$

which he referred to as a "rule of thumb" only represent the aggregate car population in the USA and that there is, in fact, a family of probability relations for the different car mass subgroups comprising the car population. As Joksch indicated this function had the undesirable property that $P(i) > 1$ when $\Delta V$ is greater than $\alpha$. On a fundamental basis this is so but when looking at the crush behaviour of cars in frontal collisions the crushing of the car front is progressive with increasing severity of crush, until, in high speed impacts, the occupant compartment is extensively crushed.
Assume that the probability relation given in equation 17 is a valid representation of actual behaviour. The theory advanced in this paper shows that injury severity is inversely related to car mass and that each car mass sub-group within a car population would have its own probability distribution and alpha value. Now taking a car population consisting of a number of sub-groups of cars of masses, $M_1$, $M_2$, etc. the overall aggregate probability of injury risk is,

$$P(i) = \sum_{c=1}^{c=n} \left( \frac{N_c}{N_e} \right) \cdot \left( \frac{\Delta V}{\alpha_c} \right)^k$$

This is equivalent to,

$$P(i) = \Delta V^k \cdot \sum_{c=1}^{c=n} \left( \frac{N_c}{N_e} \right) \cdot \left( \frac{1}{\alpha_c} \right)^k$$

The proportionality term between $P(i)$ and $\Delta V$ in equation 25 will remain constant until $\Delta V = \alpha_1$, the value for $P(i) = 1.0$ for the smallest car mass sub-group in the population. At $\Delta V$ values above $\alpha_1$ the value of the proportionality term in equation 19 will continuously decline as $\Delta V$ increases. When plotted on a log-log basis the aggregate probability-$\Delta V$ relation will take the form shown in Figure 2, a linear relation until $\Delta V = \alpha_1$ is reached followed by a curve asymptotic to $P(i) = 1$.

**Figure 2. Theoretical Injury Probability - Velocity Relationships**

**DISSIMILAR CAR COLLISIONS** We can use equation 15 to obtain a relative injury risk equation which accounts for the dynamic crush behaviour of the car population and for the
reducing effective mass of the cars with crush. In order to do so we require information about
the relative energy absorption of the respective cars in the collision pair.

These energy absorption characteristics are dependant on the force balance between the
colliding pair and in particular on the matching of the combined inertial and crumpling forces
of each car, i.e. the interface force between the car fronts. Previous analyses (Wood 1993 c,
1995 a,b) were based on matching the forces exerted on the uncrushed portions of the car
structure and not the interface forces which are the sum of the inertia and crushing forces.
When there is an offset collision or one in which the crush profile is triangular in shape,
quartered tests show that during the initial stages of impact there are negligible or very low
crushing forces imposed on the occupant compartment (O'Riordain 1994, Wood 1996). In
such circumstances the initial interface forces are the inertial forces due to the rapid
deceleration of the crumpled portions of each front structure. The presence of inertial forces
has been confirmed by comparisons of measured interface forces between car fronts and rigid
barriers and the corresponding occupant compartment forces in full barrier tests.

On the basis that these inertia forces dominate the force balancing process between the
collision pair the $V_{005}$ ratio is (see Appendix 1),

$$\frac{V_{005-c}}{V_{005-p}} = \left(\frac{M_P}{M_c}\right)^{0.3}$$  \hspace{1cm} (20)

Examination of the correlation between dynamic crush ratio using the relation in equation
10 to estimate dynamic crush from residual crush for 34 car to car collisions yields an mean
value of the exponent, $n = 0.289$, a figure close to the theoretical one of 0.3 (see Appendix
1). Substituting $n = 0.3$ into equation 15 gives the relative injury relation,

$$\text{Relative Injury Risk} = \left(\frac{M_P}{M_c}\right)^{1.55/1.70}$$  \hspace{2cm} (21)

Equation 21 shows that when the inertia forces dominate the force balance between car fronts
the relative injury risk is proportional to the power, $n = 1.55/1.70$.

Turning to the situation when the crumpling forces of each car dominate the force balance
between the car there are two approaches. Firstly, on the basis of matched impulse values,
theory shows that the ratio of energy equivalent speeds $\frac{V_{005-c}}{V_{005-p}}$ are inversely proportional
to the kerb masses of the cars. Secondly using the force balance for full width engagement of
the car fronts analysis shows (Wood 1993,c) for such circumstances that the ratio of the
absorbed energies is,

$$\frac{E_C}{E_P} = \left(\frac{M_P}{M_c}\right)^{1.4}$$  \hspace{2cm} (22)

Substitution into equation 15 for the ratio of energy equivalent speeds, $\frac{V_{005-c}}{V_{005-p}}$ yields
the relative injury relation,

$$\text{Relative Injury Risk} = \left(\frac{M_P}{M_c}\right)^{2.83/3.81}$$  \hspace{2cm} (23)

where the exponent range, $n = 2.83/3.34$ is obtained substituting the inverse of mass ratio
for the $V_{005}$ ratio while the exponent range, $n = 3.2/3.81$ derives from the full width barrier
test data. The relative injury risk relation in equation 21 applies when the inertia forces of the
crushing car fronts dominates the interface forces between the two cars, i.e. offset collisions
and low to moderate severity impacts. Equation 23 applies when the crushing forces of the structure dominate i.e. when there is extensive crushing over the width of the cars.

**COMPARISON OF THEORY WITH REAL LIFE EXPERIENCE**

**COLLISIONS BETWEEN SIMILAR CARS** Ernst et al (1991,a,b) examined the variation in the risk of serious or fatal injury with car size to drivers involved in frontal collisions between cars of similar size in Rhine-Westphalia over the period 1984 to 1988. He separately examined the risk in rural and in urban crashes. The car mass range involved in the study was 700 kg. to 1400 kg. Evans and Wasielewski (1987) carried out a similar study of driver serious or fatal injury in North Carolina and in New York State. Evans and Frick (1992) carried out an examination of driver fatalities in similar car to similar car collisions using the F.A.R.S. data. These five data sets have been analyzed by Evans (1994,b) and by Wood (1993,c). The analyses show that the injury risk, be it of serious or greater injury or of fatality on its own, is proportional to car mass ratio. It is of interest to note that the injury risk is related to mass ratio for both serious and greater injury as one group and for fatalities on their own as a second group. As these collisions involve cars of similar mass to each other it is clear that the mass of the cars has no role in causing the risk of injury. Equation 5 shows that the risk is related to two factors, the collision speed and the crush distance. All of the evidence from crash studies in various countries indicates that the range of collision speeds are the same independent of car size. This leaves only one causative factor, the crush distance of the car fronts. As shown earlier this is proportional to car length. Consequently the data indicates that the prime causative factor is car length (size) and that global injury risk can be regarded as being related to the 2.5 power of the average acceleration imposed on the car.

Figure 3. Injury Severity - Velocity Data for U.K. Frontal Collisions (Harms 1991)
SINGLE CAR COLLISIONS Evans (1984,1985) examined driver injury risk in single vehicle accidents of all types. He showed that the risk of a fatality increased by a factor of 2.4 when the car mass was reduced from 1800 kg. to 900 kg. This is in excess of the 2:1 increase predicted by theory. However the theory only directly applies to single car collisions into fixed objects which do not break nor absorb any significant portion of the cars' kinetic energy.

Jones (1988) compared single car frontal collisions into fixed objects where the configuration of impact damage was similar to that found in barrier tests. He found for restrained drivers that the probability of serious injury or of fatality was very strongly correlated with the Chest Deceleration as measured on the anthropometric dummies used in the NCAP tests carried out by NHTSA. Analysis of Jones's data for restrained drivers shows that the probability of injury is a power function of Chest Deceleration with an exponent, n, value between 2.2 and 2.7 depending on the car crush severity as measured using the TAD scale.

INJURY SEVERITY VERSUS $\Delta V$ Harms (1991) reported on the Cooperative Crash Injury Study which has been carried out in Britain for a number of years under the sponsorship of the Transport and Research Laboratory. He analyzed injury severity in terms of AIS and for frontal car collisions detailed the mean $\Delta V$ for belted front seat occupants, driver and passenger, for the different injury levels up to MAIS 6 (Unsurvivable - fatality). Figure 3 shows the data and the power regression obtained for this data. The regression is,

$$\frac{\text{MAIS}}{\text{MAIS 6}} = \left(\frac{\Delta V \text{ mph}}{40.8}\right)^{2.1} \tag{24}$$

Figure 4. U.S. Fatality Probability Data for Belted Drivers from Evans (1994).
The regression has a coefficient of determination of 0.853. The mean value of the exponent, \( n \), is 2.1 with a standard error of 0.33 giving a one standard error range of \( 1.77 \) to \( 2.43 \). The mean value of 2.1 is within the predicted range of \( 1.83 \) to \( 2.34 \) obtained from the theoretical model which includes dynamic crush and mass reduction effects and compares closely with the mean predicted value of the exponent, \( n \), of 2.08.

INJURY PROBABILITY AND \( \Delta V \) Figure 4 reproduced from Evans (1994) shows the variation in probability of fatality as a function of \( \Delta V \) for belted drivers for the U.S. based on the NASS data for the period 1982 - 1991. It is clear that the data follows the form outlined in Figure 2. The data for the linear portion of the curve comprises six data points for mean \( \Delta V \) values between 12 mph and 37 mph. Regression analysis gives the relation,

\[
P(\bar{f}) = \left( \frac{\Delta V}{55.53 \text{ mph}} \right)^{5.55}
\]  

which has a coefficient of determination of 0.996.

The \( \Delta V \) value corresponding to \( P(\bar{f}) = 0.5 \) for this U.S. data set is 49.0 mph. This is higher than the value obtained for the car population in Britain of 40.8 mph, refer to equation 24. According to the theory in this paper the difference in the two values of \( \Delta V \) for \( P(\bar{f}) = 0.5 \) is due to the differences in the mean masses of the two aggregate car populations. The theory, from equation 15, indicates that the mean mass of the U.S. car population involved in two car collisions is

\[
\bar{M}_{U.S.} = \bar{M}_{U.K.} \cdot \left( \frac{49}{40.8} \right)^{2.1}
\]

Analysis of the car population involved in two car collisions in Britain using data published by the U.K. Department of Transport (1993) shows that the mean mass of cars involved in accidents is 932 kg. (2,054 lbs). Substitution into equation 26 yields an estimate for the mean mass of the U.S. car population involved in two car collisions of 1,369 kg. (3,017 lbs). Using the one standard error range for the exponent in equation 32 of 1.77/2.43 yields an estimated mass range for the U.S. car population involved in accidents of 1,289 kg. (2,840 lbs) to 1,452 kg. (3,205 lbs.).


The estimated average weight of the U.S. car population over the period 1982 to 1991 from published data of 2775 lbs. to 3188 lbs. compares closely with the estimate predicted from equation 32 of 2,840 lbs to 3,205 lbs.

COLLISIONS between DISSIMILAR CARS

SERIOUS INJURY Ernst et al (1991,a,b) also analyzed driver serious and fatal injury frontal accidents between cars of different sizes. This study was of both urban and of rural accidents. The car sizes were classified into groups by mass. Tingvall et al (1991) have developed the Folksam paired comparison method for the evaluation of the relative safety of individual car
models in car to car crashes. It is applied to all car to car collisions including frontal crashes. Serious or greater injury are considered. The method has been used to rate the relative safety of car types under Swedish conditions. Ernvall et al (1992) have applied this method to car to car collisions in Finland. Again all car to car collisions are included in the evaluation.

Regression analysis of the four sets of data using power regression yields values for the exponent, \( n \), of 1.51 from the Tingvall (1991) data, 1.71 and 1.92 for rural and urban crashes from Ernst (1991a,b) and 1.99 from Ernvall (1992).

The field data shows that the relative risk of serious or greater injury is a power function of mass ratio (mass of partner car / case car) with the exponent, \( n \), in the range 1.51/1.99. This range compares with the theoretical prediction, based in matching of inertia forces, of 1.55/1.70.

FATALITIES Fontaine (1994) has examined car to car collisions in France. She examined the effects of case car mass, the mass of its collision partner and of vehicle performance on the risk of driver fatality in car to car crashes. In head-on and offset frontal crashes her study shows that the predominant parameters are the mass of the case car and of its collision partner. Evans (1985,'91,'92,'93) has extensively studied the role of car mass ratio on the relative risk of driver fatality, both belted and unbelted, in frontal collisions and in all car to car crashes. Analysis of Fontaine's (1994) data gives \( n = 3.2 \) while Evans's data yields values of exponent \( n \), from minimum of 2.70 (Evans, all directions belted/unbelted - USA 1980 Model Year +) to a maximum of 3.74 (Evans, frontal crashes belted/unbelted - USA). These compare with the theoretical values which are in the range 2.83/3.81.

CONCLUSIONS

The field evaluations of the car size and mass effects on relative injury risk show patterns and trends which are broadly similar to those predicted by the theory outlined here. This theory shows that the dominant causative factor of relative injury risk in collisions between pairs of similar sized cars is the size, i.e. the length of the car, all other factors being similar. The theoretical model indicates that the length of the car is the key factor is single car collisions into fixed objects. This is supported by Jones' (1988) evaluation of real life crashes of this category of single car collisions. This conclusion is predicated on the basis of similar frontal crush behaviour albeit proportional to length. For individual car types their actual crush behaviour vis a vis the car population will determine their relative safety in these types of crashes.

The theoretical relative injury severity/ \( \Delta V \) relation put forward of the behaviour of the car front structure is supported by Harms (1991) data for car to car frontal collisions in Britain. Also comparison of the U.K. and U.S. data from Evan's (1994 a) probability studies is consistent with the theoretical prediction of the mass effect in car to car collisions.

In collisions between cars of different sizes there are two major causative factors, the mass ratio of the colliding pair and their relative energy absorption properties. A subsidiary factor, the mass reducing effect of high levels of crush, also contributes to increasing the relative injury risk to occupants of the lighter car. Both the theory and the field data show that the relative energy absorption properties alter as between injury crashes and fatal crashes. The theory indicates that the inertia forces of the collapsing front structures determine the force balance and the ratio of energy equivalent speeds between the cars where the nature of the collision is such that the initial crushing forces are low. Evaluation of collision data from 34 car to car injury accidents yields correlation of crush depths in line with this theory albeit with a high degree of scatter. Also the real life injury severity comparisons match with the theoretical predictions based on the dominance of inertia forces in the car to car interface force balance.
In fatal accidents the real life relative risk correspond with the theoretical relative risk based on the full width crush energy characteristics of the car population with the smaller car absorbing a much higher proportion of the collision energy with its consequent greater crush. Not surprisingly this difference as between injury and fatal accidents does not occur in crashes between similar cars. One explanation for this difference is that the dominant factor in the matching of interface forces for injury accidents is the inertia force of the rapidly decelerating crumpled masses at the fronts of the cars while the crumpling forces dominate the fatal injury accidents. The theory also indicates that it is the energy equivalent speed $V_{eqs}$ and not $\Delta V$ which is important in terms of injury severity.

The theoretical model is based on the car population having the same non-dimensional structural characteristics and on having a unique length to mass correlation. For individual car types in car to car collisions it is their mass and crush behaviour vis a vis the car population which will determine whether they are relatively more or less safe than the norm, i.e. when cars of similar length collide together the heavier car will be the safer for similar crush characteristics and for similar length and mass the car with the stiffer structure will be relatively more safe.

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APPENDIX 1 - FORCE BALANCE BETWEEN CAR FRONTS

Consider two cars colliding together. They form a closed system and when considering the energy to be absorbed in order to stop their relative motion we are only concerned about the masses of the cars and the relative velocity of the cars one to the other. On initial contact their approach velocity is the vector difference of their individual velocities (i.e. relative or collision closing speed). This speed gives no indication as to the behaviour of the interface between the two cars, i.e. their respective car fronts or of their respective crush behaviour. However the forces at the interface are equal and opposite and comprise the sum of the forces necessary to cause each structure to collapse and the inertia forces due to the rapid deceleration of those portions of each car front in which structural collapse has been initiated. In equation form the force balance is,

\[ F_{\text{crush-c}} + F_{\text{inertia-c}} = F_{\text{crush-p}} + F_{\text{inertia-p}} \]  \hspace{1cm} (27)

When there are low crushing forces then the essential force matching is obtained by the balance of the two inertia forces. For structures of uniformly distributed mass the inertia forces are proportional to the square of instantaneous relative velocities and to masses per unit length of the structures. Matching the inertia forces we have,

\[ \left( \frac{M_c}{L_c} \right) \cdot (V_{\text{rel-c}})^2 = \left( \frac{M_p}{L_p} \right) \cdot (V_{\text{rel-p}})^2 \]  \hspace{1cm} (28)

where \( V_{\text{rel-c}}, V_{\text{rel-p}} \) are the velocities of the fronts of each of the cars relative to the respective car centre of gravity and the sum of \( V_{\text{rel-c}} \) and \( V_{\text{rel-p}} \) equals the instantaneous closing velocity of the collision pair. This equation also shows when the inertia forces dominate that the ratio of the energies dissipated by each car during mutual crushing are proportional to their respective lengths. Also \( V_{\text{rel-c}}, V_{\text{rel-p}} \) are the same as the energy equivalent speeds, \( V_{\text{ees-c}}, V_{\text{ees-p}} \). Considering the matching of the inertia forces alone the ratio of the relative deformation or crumpling velocities, i.e. the ratio of energy equivalent speeds is,

\[ \frac{V_{\text{ees-c}}}{V_{\text{ees-p}}} = \left( \frac{M_p}{M_c} \right)^{\frac{1}{2}} \]  \hspace{1cm} (29)

Substituting the Length to Mass power relation from equation 4 gives,

\[ \frac{V_{\text{ees-c}}}{V_{\text{ees-p}}} = \left( \frac{M_p}{M_c} \right)^{0.3} \]  \hspace{1cm} (30)

This equation shows when considering inertia forces alone that the energy equivalent speeds are proportional to the inverse of mass to the power of \( n = 0.3 \). In practice while in offset collisions the inertia forces dominate the force balance during the early stages of crushing these force diminish with the square of instantaneous speed while at the same time the crushing forces increase and dominate towards the latter stages of mutual crushing.

As the impact duration is common then the respective centre of gravity displacements are proportional to the relative crushing speeds, \( V_{\text{rel-c}}, V_{\text{rel-p}} \) and when the inertia forces dominate the force balance between the cars, the relative energy equivalent speeds, \( V_{\text{ees-c}}, V_{\text{ees-p}} \).
where $d_c$ and $d_p$ are the respective relative displacements (crush) of the car fronts with respect to their centres of gravity. Wood (1992c) examined the relationship between crush ratio and the mass and length ratios for offset frontal collisions. He found that the empirical relation for residual crush ratio was,

$$\frac{V_{e_{es-c}}}{V_{e_{es-p}}} = \frac{d_c}{d_p}$$

This analysis was based on a sample of 34 frontal collisions where both cars were examined and measured and where examination showed that the collision was one where the relative motion at the interface between the colliding pair ceased at maximum crush i.e. the collision was not a sliding type impact. While there was very substantial scatter in the collision data the regression was statistically significant at the 2.5% level. This regression is based on measurements of the residual or permanent crush to the cars which is on a surrogate for the dynamic crush. When the dynamic crush is estimated using equation 16 the data for the 34 collisions gives the power regression,
\begin{equation}
\left( \frac{d_c}{L_c} \right) = \left( \frac{M_p}{M_c} \right)^{1.1486} \left( \frac{L_c}{L_p} \right)^{1.1486}
\end{equation}

is obtained. It has a correlation coefficient of $r = 0.4$ which is significant at the 2.5% level. The exponent, $n$, which has a mean value of 1.1486, has a standard error, S.E. = 0.465. The data and the regression are shown in Figure 5. Substituting from equation 4 for the car length/mass power relation and using the mean value for the exponent, $n$, yields the relationship,

\[ \frac{V_{ees-c}}{V_{ees-p}} = \frac{d_c}{d_p} = \left( \frac{M_p}{M_c} \right)^{0.289} \]

The exponent value of 0.289 is remarkably close to the theoretical relation obtained by matching the inertia forces alone. The +/- one standard error range for the exponent in equation 40 is 0.01/0.568. The range of scatter is a measure of the complexity of the process of mutual crushing between cars and of the manner in which this will vary with collision type, full width, offset, etc. and with severity.