Mathematical Modelling of the Human Cervical Spine: A Survey of the Literature

Marko de Jager
Eindhoven University of Technology, Faculty of Mechanical Engineering,
P.O.Box 513, 5600 MB Eindhoven, The Netherlands.

Abstract

This paper summarizes the results of a literature survey concerning the aspects relevant for mathematical modelling of the human cervical spine. Both relatively simple two-pivot models and the more detailed discrete parameter and finite element models describing head-neck dynamics are reviewed. Further, attention is given to data on the physical properties of cervical spine components and to experiments which can be used to validate a cervical spine model.

It is concluded that a number of sophisticated models are available in the literature. However, these models do not simulate the relative head-motion better than two-pivot models do. Furthermore, it is concluded that data on the material characteristics of cervical components are incomplete as well as data for a detailed validation of local vertebral movements. Recommendations for additional experimental research are given.

1 Introduction

Epidemiologic studies have shown that injuries to the cervical spine are quite commonly found in traffic accidents (mainly car crashes), see e.g. Refs. [8,19,45,57,74]. The mechanisms of injury to the cervical spine are not fully understood, because the human cervical spine is a mechanically complex structure and subjected to a wide variety of (traumatic) loading conditions in accidents.

Both mathematical modelling and experimental research are used to study the mechanical behaviour of complex, biological systems. A mathematical model can be used to simulate the behaviour of the system in different (experimental) situations and can be used to obtain information that cannot be obtained from experiments. Mathematical modelling the mechanical behaviour of biological systems has proven to be a valuable tool in other fields of research as well as in the research on spine biomechanics, see e.g. Refs. [31,39,40,47,77,84].

According to Ward and Nagendra [77], the major pitfalls in mathematical modelling of biological systems are: oversophistication, lack of good physical properties data, and lack of validation. Oversophistication will result in a model including too many details of which the effect on the behaviour of the model will be difficult to retrieve. During the process of modelling, usually numerous assumptions and simplifications have to be introduced, partly due to the lack of reliable physical properties data and partly to reduce the complexity of the model. To check on the assumptions used, a model has to be validated. Validation is achieved by correlating numerical predictions with experimental results covering the situations for which the model will be used.

Goal of our study is the development of a mathematical model of the human cervical spine that incorporates injury mechanisms. This model must be able to describe, in detail, the biomechanical response of the human head and neck to various impact situations. As part of this study, a literature survey has been conducted concerning the aspects relevant for mathematical modelling of the human cervical spine. This paper summarizes the results of this survey. Not included here is a review on injury mechanisms. Valuable retrospective studies on injury mechanisms include the Refs. [30,38,50,67,78].

2 Physical Properties of the Cervical Spine

Qualitative descriptions of the biomechanical behaviour of the cervical spine are sufficiently available in the literature, e.g. Refs. [29,34,37,38,56,67,68,78]. However, only a few studies describing quantitative biomechanical aspects important in modelling the cervical spine, i.e. physical properties, have appeared in the literature. These studies will be reviewed here.

The cervical spine is an articulate structure made up of joints, allowing for motion of the head relative to the torso. The mechanically relevant components of the cervical spine are the vertebrae, intervertebral discs, facet joints, ligaments and neck muscles.

The cervical spine comprises seven vertebrae (numbered C1 through C7), which are joined by soft tissues: intervertebral discs, facet joints, ligaments and muscles. Facet joints and intervertebral discs carry load from one vertebra to another, whereas ligaments and neck muscles stabilize the cervical spine.

In studies on the biomechanics of the spine, motion segments are often used. A motion segment comprises two adjacent vertebrae together with surrounding soft tissues: intervertebral disc, facet joints and ligaments. It is the smallest unit exhibiting biomechanical features similar to the entire spinal column, which may be considered as a structure composed of motion segments connected in series. The total behaviour of the (lower cervical) spine, then, is a composite of individual motion segment behaviour. Due to its functional arrangement, the occipito-atlanto-axial joint is usually treated as a single biomechanical unit and, similar to motion segments, subject of biomechanics studies.

Since the behaviour of motion segments is dependent upon the behaviour of its components, these components should be studied too. Thus, to study (and model) the biomechanics of the cervical spine quantitatively, physical properties of both motion segments and components are needed. Physical properties include the geometrical, the inertial and the material characteristics.

2.1 Geometrical Characteristics

Geometrical characteristics include (1) the dimensions of vertebrae, articular facets, discs, ligaments, and muscles; (2) the locations of the places where the soft tissues are attached to the vertebrae; and (3) the compound configuration of all elements. Much of this information may be collected from detailed (cross-sectional) anatomy books, X-rays photographs, computed tomography scans (CT-scans), magnetic resonance imaging (MRI's) and skeletal material.

Francis [20,21] studied (variations in) the dimensions of cervical vertebrae and articular facets from human skeletal material of young adults. Nissan and Gilad [55] reported average dimensions for the mid-sagittal appearance of vertebrae, which is idealized by a square-box approximation for the vertebral body to which a triangular shaped arch is attached. Parameters for vertebrae C2-C7 were measured from lateral X-rays of numerous male cervical spines. Liu *et al.* [42] determined the geometry of all cervical vertebrae of young males by measuring the coordinates of premarked points (36 per vertebra) relative to the vertebral body centre of mass. Furthermore, the orientation of the articular facets and the articular facet-to-centre area ratios were obtained. The data on the measured coordinates are given in Ref. [43]. Recently, Panjabi *et al.* [60] determined the linear dimensions, angulations and surface and cross-sectional areas of most vertebral components from three-dimensional coordinates of points on cervical vertebrae (C2-C7). Included are, among others, the dimensions of the vertebral body, spinal canal and pedicles. However, no measurements regarding articular facets dimensions were reported. They noted that their results agree well with those from Francis, Nissan and Gilad, and Liu *et al.* Goel *et al.* [24] obtained, for three cadavers, the origins and insertions of all the major muscles of the head-neck region with respect to both anatomical and global reference planes. They also measured weight, natural length and maximum width of each muscle.

Methods to reconstruct the three-dimensional geometry of (lumbar) vertebrae have been developed by Lavaste *et al.* [41] and Breau *et al.* [7]. The method of Lavaste *et al.* needs six geometrical parameters which can easily be obtained from lateral and frontal X-rays. With these parameters, the other dimensions of the vertebrae are calculated to reconstruct its geometry. The method of Breau *et al.* uses CT-scans to reconstruct the geometry of vertebrae.

2.2 Inertial Characteristics

Inertial characteristics include mass, location of centre of gravity, and (principal) moments of inertia of head, neck and vertebrae. The characteristics assigned to the vertebrae should represent those of a complete neck.

Data on the inertial characteristics of the head and the head and neck, have been reported by Beier *et al.* [1] and Walker *et al.* [76]. Liu *et al.* [44] determined the inertial properties of horizontal segments of a human cadaver. Since each segment contained one vertebra, the properties assigned to the vertebrae are those of neck segments at the level of the vertebrae. Liu *et al.* reported that large errors were encountered in the data for the cervical segments. Hence, their results should be interpreted with care.

2.3 Material Characteristics

Material properties are specified by constitutive equations. The unknown parameters of these equations have to be estimated from experimental results to obtain a valid model for specific material behaviour. Experimentally, material behaviour is presented by means of force-displacement (or stress-strain) curves, stiffnesses, load and deformation at failure, and so forth.

Force-displacement curves for biomechanical structures as motion segments or ligaments qualitatively have the nonlinear, sigmoidal shape depicted in Figure 1. The curve starts with a neutral zone in which little force is needed to deform the structure. At the end of this zone, the stiffness increases substantially. The stiffness usually remains fairly linear up to failure, which is identified as a (sudden) substantial drop in force. The load and deformation at this point of failure is then defined as failure strength of the structure.

Although stiffness is easily defined as the ratio of force on and (related) deformation of the structure, the nonlinearity of material behaviour gives rise to difficulties. In most publications, the experimentally obtained force-displacement curves are not reproduced, but represented by a stiffness coefficient. However, the load-displacement curves are nonlinear and thus difficult to describe by just a stiffness coefficient. Moreover, different stiffness calculations have been used by different authors. For example, for the curve given in Figure 1 stiffnesses have been calculated as: (1) the ratio of (maximum) applied force and deformation at this force; (2) the ratio of (maximum) applied force and deformation at this force minus the neutral zone deformation; (3) slope of the most linear part of the curve; (4) stiffness calculated from linear regression analysis of the curve; or (5) slope of the curve at a certain load (or deformation). Stiffness calculated by (2) or (3) is usually named "average stiffness". Calculation (5), the exact definition of stiffness for a point of the curve, may be used to represent a complete curve if stiffnesses are given for a sufficient number of points. For all definitions, the load at which or the loading range for which the stiffnesses were calculated should be given.

Material Characteristics of Cervical Spine Segments

Up to 1983, three-dimensional studies on the biomechanical properties of spine segments had been limited to the thoracic and lumbar regions [61]. To date, biomechanical properties of cervical spine segments have been examined by various investigators. In most studies, (motion)

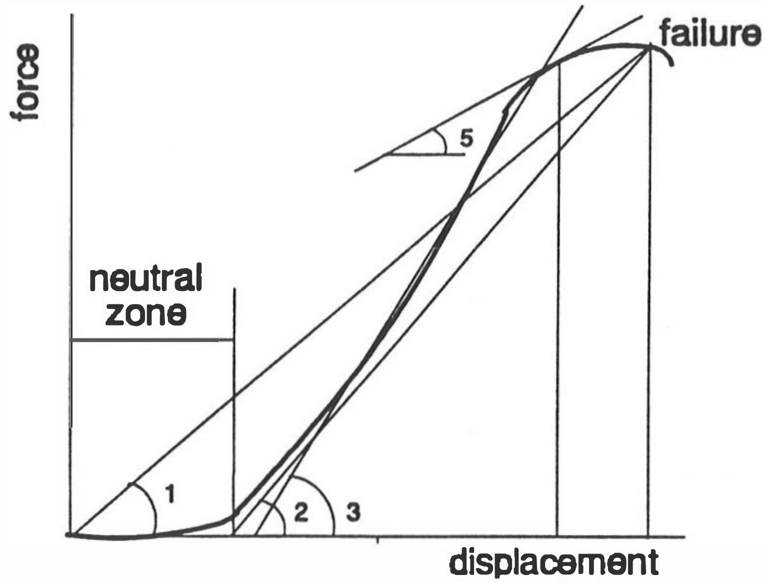

Figure 1: Typical force-displacement curve.

segments of the lower and upper cervical spine have been used to characterize experimentally the biomechanical behaviour of the cervical spine. In general, the experimental procedure is as follows. The lower vertebra is fixed and loads are applied to the upper vertebra while the resulting three-dimensional displacements are measured. Loads are applied statically (incrementally), quasistatically (low deformation rate) or dynamically. Motion segment stiffness is then calculated from the measured force-displacement curves. Unfortunately, different authors may use different stiffness calculations, which complicates a good comparison between reported stiffness values.

White and Panjabi [78] collected average stiffness coefficients of a "representative" motion segment of all regions of the spine for all modes of loading. Lower cervical spine studies have been conducted by Panjabi *et al.* [61], Moroney *et al.* [53] and Shea *et al.* [70], while the biomechanical properties of the upper cervical spine have been studied by Panjabi *et al.* [58] and Goel and co-workers [9,23,25]. These studies are described below.

Lower Cervical Spine Studies Panjabi *et al.* [61] subjected (18) motion segments to six types of translational force: right and left shear, axial compression and tension, and anterior and posterior shear. Moroney *et al.* [53] tested (35) motion segments in compression, shear, flexion, extension, lateral bending, and axial torsion. The latter used intact segments and disc segments which had their posterior elements (arch with ligaments) removed. They also measured the moments required for failure of the segments in flexion, extension or right lateral bending. In both studies, load is increased incrementally up to some maximum force (or moment) small enough to prevent injury (less than 80N or 2.2 Nm). Three-dimensional displacements of the upper vertebral body were measured after each loading step. Large variations in the (static) mechanical properties of the segments were observed, but a systematic variation of motion segment stiffness with vertebral level was not found. Therefore, average linear stiffnesses (or flexibilities) were calculated for each type of loading.

Shea *et al.* [70] subjected (27) spine segments to combinations of sagittal loads. The segments consisted of three adjacent vertebrae and the interconnecting ligaments and discs. The lower vertebra was forced to move, while displacements of the middle vertebral body were measured. Forces were measured at the upper vertebra, which was rigidly attached to a load cell. Segments were first tested non-destructively to obtain load-displacement curves, and then loaded to failure in flexion or in flexion-compression. Quasistatic loading was applied with rates up to 5 mm/s

Table 1: Average stiffness coefficients for lower cervical spine motion segments (C2-T1).

Loading type	White & Panjabi [78]	Panjabi *et al.* [61] (1)	Moroney *et al.* [53] (2)	Shea *et al.* [70] (3)	Shea *et al.* [70] (4)
(N/mm)					
tension	53	53	—	433	193
compression	200	141	1318	718	957
anterior shear	50	753	131	183	123
posterior shear	53	34	49	162	114
lateral shear	53	53	119	—	—
(Nm/degree)					
flexion	0.43	—	0.43	1.13	—
extension	0.73	—	0.73	1.88	1.74
lateral bending	0.68	—	0.68	—	—
axial rotation	1.16	—	1.16	—	—

(1) reciprocal of reported average flexibility coefficients; range of loads for which coefficients were calculated is 12-38 N
(2) range of loads is 49-74 N for compression, 10-39 N for shear, and 1-2.2 Nm for moments
(3) stiffnesses calculated at 300 N compression-tension, 150 N shear, and 5 Nm flexion-extension
(4) stiffnesses calculated at 500 N compression, 100N tension or shear, and 3.5 Nm extension

translation or 5 deg/s rotation. The load-displacement curves were nonlinear for even small applied loads: stiffnesses are low near zero displacement and increase drastically before reaching failure. The loads applied in this study were substantially higher than those in the studies of Panjabi *et al.* and Moroney *et al.* and the calculated stiffnesses (defined as the slope of the curve at a specific, relatively high, load) were also higher. Further, Shea *et al.* found that the C2-C5 region is significantly stiffer in compression and extension than the C5-T1 region.

Average stiffness coefficients reported in the studies mentioned above are summarized in Table 1.

Upper Cervical Spine Studies Panjabi *et al.* [58] studied the three-dimensional physiologic motions of the C0-C1 and C1-C2 joints. Cervical spine specimens were loaded at the occiput in flexion, extension, left and right lateral bending and left and right axial rotation. Loads were incrementally applied up to maximum moment (1.5 Nm). All six motion components of vertebrae and occiput were measured after each load step. They reported mean values for the neutral zone and range of motion, and average linear flexibility coefficients for both joints.

Goel *et al.* [23] conducted static experiments on C0-C5 specimen to quantify the moment-rotation relationship of the ligamentous occipito-atlanto-axial joint. Motion of vertebra C5 was fully restricted, while loads (pure moments) were incrementally applied at C0 up to maximum moment (0.3 Nm). Loads were applied in flexion, extension, left and right lateral bending and left and right axial rotation. The three-dimensional positions of C0, C1, C2 and C3 at no-load, after each loading step and after removing the final load were measured. Relative rotation between C0-C1, C1-C2 and C2-C3 and average nonlinear moment-rotation curves are reported.

Goel and co-workers [9,25] quantified the quasistatic and dynamic response of the occipito-atlanto-axial joint to axial rotation. They used C0-C2 specimen of which C2-motion was fully restricted. Axial torques were applied at C0 until failure of the specimen. Specimens were subjected to loading rates of 4, 50, 100 and 400 deg/s respectively. The moment-rotation curves were highly nonlinear, showing less resistance in the initial phase, followed by a sharp increase in resistance in the final phase up to failure. Average torque-rotation curves for all loading rates are given. They found that the stiffness of the specimens increased at higher loading rates, and that the angular rotation at failure did not show any significant variation with loading rate.

Material Characteristics of Cervical Spine Components

Yamada [82] reported strength characteristics of numerous biological materials (organs and tissues) obtained from human and animal cadavers. With respect to the locomotor system, mechanical properties of bone and vertebrae, cartilage, intervertebral discs, ligaments, muscles and tendons are given. Yamada's data on vertebral bodies, intervertebral discs and a few other tissues relevant with respect to the spine have been reproduced in Ref. [68]. Fung [22] used the principles of continuum mechanics to describe the mechanical behaviour of bio-solids (hard and soft tissues) and bio-fluids. Emphasis is put on constitutive equations that can be used to describe the behaviour of biological tissues. Constitutive equations for, among others, muscles, bone and cartilage are presented.

Vertebrae, intervertebral discs and facet joints Yamada reported failure strength and deformation data for cervical vertebrae and discs subjected to compression or tension. Stiffness data for cervical vertebrae have not been reported to date. Disc stiffnesses have been reported by Moroney *et al.* [53] (compression) and Pintar *et al.* [62] (tension). Since the deformation of vertebrae is small compared to the deformation of discs, vertebrae may be treated as rigid bodies. Vertebral deformation may be taken into account by transferring it to the disc characteristics. To date, biomechanical properties of cervical facet joints have not been reported in the literature, although the capsular ligaments of these joints have been studied.

Ligaments Force-displacement curves for spinal ligaments typically have the nonlinear, sigmoidal shape, represented in Figure 1. Average failure strengths of the most important upper and lower cervical spine ligaments have been collected by White and Panjabi [78]. Chazal *et al.* [10] studied the geometrical and biomechanical properties of various spinal ligaments subjected to quasistatic loading (1 mm/min). With respect to the cervical spine, data were obtained for the anterior and posterior longitudinal ligaments. Included are mean values for stresses and strains at three characteristic points of the sigmoidal force-displacement curve. Dvorak *et al.* [15] reported the failure strength and the dimensions of the alar and transverse ligaments of the upper cervical spine. The ligaments were loaded quasistatically at a rate of 1.5 mm/s.

Myklebust *et al.* [54] reported average values for the failure strength of spinal ligaments for all spinal levels. Ligaments were tested *in situ* by sectioning all elements except the one under study. Load was applied quasistatically (1 cm/s) until failure of the ligament occurred. Force-displacement curves typically exhibited a sigmoidal shape. With respect to the cervical spine, data for the most important ligaments are reported. Yoganandan *et al.* [83] investigated the *in situ* dynamic response of the anterior longitudinal ligament and the ligamentum flavum of the cervical spine. Four different loading rates (9, 25, 250, and 2500 mm/s) were applied to obtain the (nonlinear) displacement-force curves up to failure. Both failure strength and stiffness (slope of the most linear part of the response) of the ligaments have been reported.

3 Mathematical Models of the Human Cervical Spine

Injury analysis requires that the mechanical behaviour of the cervical spine is represented in detail. In other words, the model must not only describe the *global* kinematics and dynamics of the head and neck, but also the *local* kinematics and dynamics of individual vertebrae and other relevant cervical components.

In the literature, four types of models describing head-neck dynamics are found: two-pivot models, continuum mechanics models, discrete parameter models, and finite element models. Pivot models are already capable of describing the global motion of the head and neck relative to the torso, but cannot describe the mechanics of the neck in detail. In continuum mechanics models, the cervical spine is represented as a homogeneous beam column, that is both geometry and mechanical behaviour are strongly simplified. Both discrete parameter and finite element models allow for a more detailed representation of the mechanical behaviour of the various anatomical structures of the human neck.

In discrete parameter models the spine is idealized as an assemblage of individual rigid vertebrae, connected by massless spring and damper elements representing the intervertebral disc and the surrounding soft tissue complex, and (sometimes) the muscles. Mass and inertial properties of the system are lumped into the rigid vertebrae. Discrete parameter models cannot completely quantify the mechanics of the spine, because of the complex geometry and material behaviour and the nonlinear mechanical response of the spine. To overcome (a part of) these limitations finite element models were introduced, which allow for a more detailed description of the mechanics of the spine.

In finite element models the spine is also considered as a structure formed by various anatomical components, but now each component is broken down into a large number of deformable elements which are in contact with each other. Each element has the continuum material properties of the anatomical component it belongs to. The mass is concentrated in the corners or along the sides of the elements. In principle, the finite element method can accommodate any type of geometry, loading, material behaviour and boundary condition data.

The major drawback of finite element modelling is that it yields complex models (with many parameters) that are computationally inefficient and more difficult to validate compared to discrete parameter models. In fact, discrete parameter models are a subset of finite element models: a subset of simplified models with fewer degrees of freedom and, therefore, fewer equations. This allows that nonlinearities are easier handled and that simpler solution methods may be employed. Thus both discrete parameter and finite element models may be suited to describe the mechanics of the spine in detail.

An elaborate description of continuum models and two-dimensional discrete parameter models is provided by Yoganandan *et al.* [84], who extensively reviewed mathematical models of the spine and spinal components (up to 1986), so these models are not included here.

3.1 Two-Pivot Models

The global head motion (motion of the head relative to the torso) can already be described by relatively simple three-segment, two-joint models. In these *two-pivot models*, the neck is modelled as a rigid or extensible link that connects the movement of the torso (at T1) to the head. Head and torso motion is determined from experiments (sled tests) with volunteers or cadavers. The experimental obtained torso motion is used in the model to predict head motion (angular and linear displacement, velocity and acceleration). This head motion is then compared with the experimental head motion to validate the model. Pivot models have been developed by various authors, among others, by Bosio and Bowman [6], Tien and Huston [73] and Wismans *et al.* [80, 81]. From their results it can be concluded that two-pivot models are indeed capable of simulating global head behaviour quite accurately. However, a two-pivot model suited for all impact directions and impact levels could not be obtained thus far. For details, the reader is referred to the original publications.

3.2 Three-Dimensional Discrete Parameter Models

The first three-dimensional discrete parameter models of the spine were developed by Panjabi [59] and Belytschko *et al.* [4]. Both took the human spine to illustrate their general methods for the construction of three-dimensional discrete parameter models and the determination of the governing equations of motion. The elements used are rigid bodies connected by deformable elements (springs and dampers).

Chen [11] developed a three-dimensional model of the human ligamentous spine suitable for use in both static and dynamic loading situations. Included are rigid vertebral bodies, deformable discs and posterior spinal elements (facet joints and ligaments), and the initial curvature of the spine. Chen reduced the model to two dimensions to analyze the pilot ejection problem.

Huston *et al.* [32,33] developed a head-neck model to predict head motion in impact situations. The model comprises nine rigid bodies, representing torso, cervical vertebrae and skull, connected by intervertebral discs, ligaments and muscles. Discs and muscles are modelled as (visco)elastic solids, ligaments as nonlinear elastic bands. Both muscles and ligaments exert force only in tension. One-way dampers are used to model joint constraints which limit the relative motion of the bodies. Tien and Huston [72] simplified this model by taking an overall representation of the force-displacement behaviour of the soft tissue complex (disc, muscles and ligaments). They used empirical expressions for the forces and moments exerted by the soft tissues on the vertebrae. This yielded a computationally more efficient model with less parameters. Values for the (stiffness and damping) parameters were obtained from curve fitting of model prediction with experimental results of volunteer sled tests, and (hence ?) a good match between the numerical and experimental found head acceleration and velocity was obtained. The resulting model was further simplified by Tien and Huston [73], who fused the cervical vertebrae into a single rigid body, which resulted in the two-pivot model mentioned earlier. Schneider *et al.* [69] added a moveable rigid jaw to the model of Tien and Huston [72], to investigate the dynamic response of the jaw during whiplash (extension). The jaw-head joint allowed for both rotation and translation during jaw opening.

Suh [71] describes a method to construct a dynamic model of the cervical spine. No quantitative data of the model are given, because not enough data on material properties were available at that time. Skull and vertebrae are modelled as rigid bodies. Ligaments and muscles in passive mode are modelled as nonlinear spring-dampers; and muscles in active mode as force generating elements. Facet joints are modelled with nonlinear spring-dampers that are compliant in tension and stiff in compression. To simulate disc behaviour, it is assumed that the overall effect of a complex (combined) displacement is the sum of the independent displacements for which force-displacement characteristics were measured.

Merrill *et al.* [26, 51] extended the two-dimensional discrete parameter model of Reber and Goldsmith [66] into three dimensions. The resulting model was further improved by Deng and Goldsmith [12,13]. This lumped parameter model of head, neck and upper torso comprises ten rigid bodies representing torso with T2, the vertebrae T1 through C1, and the head; see Fig. 2. The overall mechanical response of intervertebral discs, ligaments and articular facets is lumped into a linear stiffness matrix, relating force and moment to translation and rotation. The off-diagonal elements of this matrix represent coupling of motion in one direction with load in another direction. Intervertebral damping is represented by linear damper elements. The model incorporates fifteen pairs of neck muscles, but only for the passive state. Muscles are represented by three-point spring elements with nonlinear constitutive relationships. To validate the model, numerical predicted head kinematics were compared to those obtained from frontal and lateral volunteer sled acceleration tests. Qualitatively, the response patterns were in reasonable agreement. Quantitatively however, correspondence is less good: especially the head accelerations remain well below those from the experiments during the initial impact phase. Finally, Luo and Goldsmith [46] extended the model of Deng and Goldsmith to include the lower torso. The model comprises ten rigid bodies representing the head; the vertebral pairs C1-C2, C3-C4, C5-C6, C7-T1; the entire thorax; the lumbar vertebral combinations L1-L2, L3, L4-L5; and the pelvis.

Finite Element Models

Belytschko *et al.* [2,3] developed a three-dimensional finite element model of the head-spine-torso structure to study the pilot ejection problem. The model includes the complete spine, pelvis and skull and may also include the rib cage. Rigid vertebrae are connected by discs, ligaments and articular facets, which are represented by several deformable elements which collectively provide resistance against axial, torsional, bending and shear loads. Although a more detailed

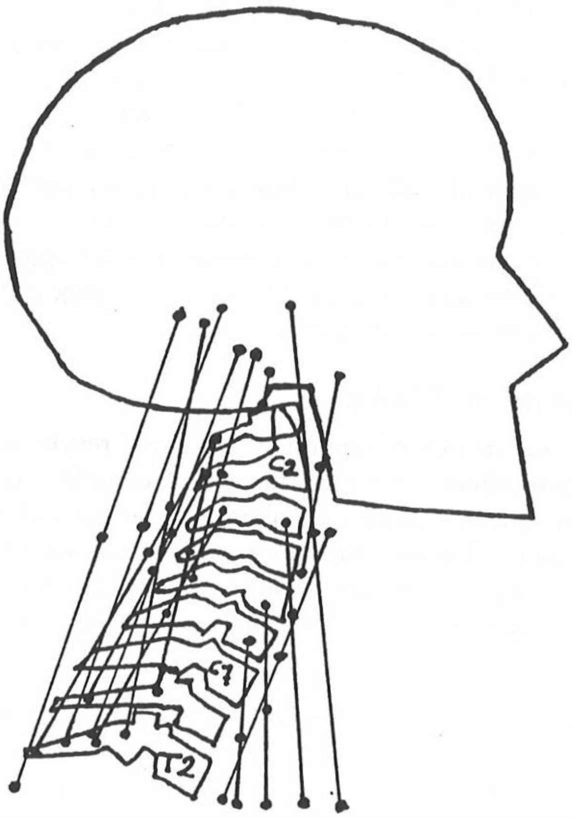

Figure 2: The model of Deng and Goldsmith (Redrawn from Ref. [13]).

representation of the neck is available within the model, only simulations with a simplified (beam element) representation of the cervical spine are reported. The model has been advanced recently by Privitzer and Kaleps [65] to study the effect of head-mounted systems on the dynamic response of the head and spine.

Williams and Belytschko [79] used the approach of Belytschko *et al.* to develop a detailed head-neck model. The model comprises rigid vertebrae T1 through C1 and the skull connected by deformable elements representing discs, facet joints, ligaments and muscles. Beam elements with linear torsional and bending stiffnesses and bilinear axial stiffness are used to represent intervertebral discs (C2-T1) and connections between C2-C1 and C1-C0. Beams between C2-C1 and C1-C0 were arranged differently to account for the unique properties of this region. Articular facets are represented by a special shaped continuum element, with axial and shear stiffnesses. Ligaments are represented by nonlinear spring elements. Twenty-two different neck muscle groups are included. Muscles are represented by spring elements the axial force of which may be activated independently of the elongation to mimic muscle contraction. These elements include intermediate sliding nodes so that the muscles can curve around bones. The model was validated for frontal and lateral impact accelerations. Simulations in which the muscles are passive throughout the simulation and in which the muscles start contracting after some time are compared to the experimental results. For frontal impacts, predicted head kinematics agree well with the experimental results, whereby the model with muscle contraction gives slightly better results than the passive muscle model. For lateral impacts, correspondence is less good, showing substantial deviations between numerical and experimental (maximum) head acceleration and displacements.

Hosey and Liu [27, 28] developed a three-dimensional finite element model of the head and neck. The model was developed primarily to study the mechanics of head (skull and brain) injury. It incorporates skull, dura, cerebrospinal fluid space, brain, jaw, cervical vertebrae and discs, and spinal cord. Each vertebra and each disc is modelled as a single element. Since the formulation is linear, the model is restricted to small displacements and rotations.

Dietrich *et al.* [14] describe a three-dimensional finite element model of the human spinal system. The model includes the spine (vertebrae C3-L5), sacrum, pelvis and ribcage, modelled as rigid bodies. They omitted atlas and axis because of the different function and shape of these vertebrae. The soft tissue components are modelled with deformable finite elements. Basic ligaments of the spine and important muscles that influence behaviour of the spinal system are included too. External forces (static load or inertial forces) can be applied to the model. The model allows for both static and dynamic analysis of forces occurring in the spinal system. An example of a static analysis is included in the paper.

4 Validation of Mathematical Models

A model is validated through comparison of numerical predicted results on head-neck responses to impacts with similar results obtained from experiments. A complete and thorough validation of a detailed model should include a comparison of results on both the global and the local dynamics and kinematics of the head-neck structure in various impact situations. Global refers to the forces on the head, neck and torso and to the motion of the head relative to the torso (T1). Local refers to the forces acting on each cervical component at each vertebral level and to the motion of each vertebra.

Global Validation For global validation, the results of sled acceleration tests can be used. These tests have been performed with human volunteers and cadavers primarily to obtain the head-neck response to impact accelerations when direct head impact is not involved. Well known are the sled tests performed with volunteers at the Naval Biodynamics Laboratory (NBDL) in New Orleans, Louisiana, and with cadavers at the University of Heidelberg in Germany. Results of these and other sled tests have been analyzed and are reported in the literature e.g. Refs. [5, 16–18, 35, 36, 48, 80, 81].

The overall dynamics (angular and linear accelerations) of the head and neck are obtained from accelerometers mounted to the subject's head and thorax (at T1), while the overall kinematics are obtained from high speed film-recordings taken during the impact. Volunteers have been subjected to moderate (non-injurious) impact levels, whereas cadavers have been subjected to moderate – for comparison with volunteer-tests – and severe (injurious) impact levels. Accelerations may be applied in several directions: frontal, lateral, oblique, rear-end and vertical (pilot ejection). However, no results of experiments with rear-end impacts that give information about the global dynamics and kinematics have been reported in the literature. For volunteers these data are, indeed, difficult to obtain due to the vulnerability of the neck for rear-end impacts.

Local Validation For local validation, a detailed knowledge of the dynamic and kinematic response of the human cervical spine is needed. Most of this knowledge cannot be determined experimentally from volunteers and is even hard to obtain from experiments with cadaveric material. As a consequence, only few experiments that may give such results have been reported in the literature.

For quasistatic and dynamic axial compression of the straightened cervical spine, Yoganandan, Pintar and co-workers [63, 64, 85, 86] obtained the sagittal plane movements of vertebrae and occiput from film-recordings of markers placed in these bony parts. Detailed results on these movements are given.

Studies that may give the localized movement of vertebrae in quasistatic, voluntary (muscle induced) motion of the head include the following. Moffat and Schultz [52] and Van Mameren [75] conducted X-ray studies on the sagittal plane motion (patterns) of cervical vertebrae for voluntary flexion and extension movements. Margulies *et al.* [49] used magnetic resonance imaging to measure the *in vivo* motion of the cervical spinal cord in human volunteers for stepwise flexion and extension of the neck. From these images vertebral movements can be obtained too.

5 Discussion and Conclusions

Goal of our study is the development of a mathematical model of the mechanical behaviour of the human cervical spine. The model must be able to describe the biomechanical response of the human head and neck to various impact situations (various directions and magnitudes of impact forces). Furthermore, to incorporate injury mechanisms, the model must describe the local kinematics and dynamics of individual vertebrae. We reviewed the literature concerning physical properties and mathematical models of the cervical spine and data for validation of the model. From this study, the following conclusions can be drawn.

Material characteristics From the literature, geometrical and inertial characteristics are sufficiently available. However, quantitative information on the mechanical behaviour of cervical components and motion segments remains incomplete. Fortunately, much progress has been made in recent years. For some cervical spine components material characteristics are not available at all, while for other components data on the material characteristics are incomplete with respect to types (quasistatic, dynamic) and directions of loading.

Experiments have been conducted to obtain material characteristics for both upper and lower cervical spine segments. With respect to lower cervical spine segments, experiments were conducted with either static or quasistatic loading. Only in a few of these studies, loads were applied up to failure of the specimens. With respect to the upper cervical spine, data from static experiments (with low maximum load) for various types and combinations of loading have been reported. Results for quasistatic and dynamic applied loads (up to failure) have been reported for axial rotation only.

Mathematical models A number of sophisticated models describing the head-neck dynamics have been reported in the literature. These include the discrete parameter models of Deng and Goldsmith, and Tien and Huston and the finite element model of Williams and Belytschko, which is the most detailed model. These models have been validated, but only for a small number of impact situations and only for global (head) kinematics. Model predictions were compared to the results of volunteer sled tests. In general, the numerical and experimental response patterns showed good correspondence. Quantitatively, correspondence was usually less good and substantial deviations were reported. It should be noted that only a very limited amount of physical properties data on cervical (motion) segments was available at the time the models were developed. Properties were either tuned until the model showed reasonable behaviour or estimated from data obtained from non-cervical components. More accurate data may improve the model predictions.

When the performance of these detailed models is compared to the performance of two-pivot models, it appears that the detailed models do not simulate the global movements more accurately than the two-pivot models. Thus, detailed models and pivot models seem to be equally capable of simulating the global behaviour of the head-neck system.

Validation For global validation, sufficient data are available on head-neck responses for various impact directions in which no head impact is involved, except for rear-end impacts (extension). Localized kinematics may be obtained from the results of axial compression tests or from the quasistatic volunteer tests, but these experiments cannot be compared to any of the (dynamic) sled tests.

Thus, no detailed results of experiments comparable to the volunteer and cadaver sled tests have been reported. Obviously, there is need for experiments yielding the local kinematics (and dynamics) of the head-cervical spine structure in impact situations comparable to the sled tests performed with cadavers and volunteers. Results of these experiments together with those from sled tests may then serve as a database for validation of cervical spine models.

In conclusion, two important problems in mathematical modelling the mechanical behaviour of the cervical spine are (1) the incompleteness of experimental data on physical properties, needed to develop a detailed model, and (2) the incompleteness of experimental data, needed to validate a model thoroughly. Additional experimental research on these subjects is highly recommended.

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