A DEVELOPMENT OF APPROXIMATE IMPEDANCE FUNCTIONS TO ESTIMATE GENERAL HUMAN HEAD IMPACT RESPONSE FOR OFF-AXIS IMPACTS

by

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ABSTRACT:

Mathematical functions to estimate indirect head impact responses were developed in this paper. Experimental human cadaver sinusoidal head impact responses in the A-P (frontal), L-R (lateral), P-A (rearward) and S-I (superior-inferior) directions were used as reference experimental bases for the generation of the "off-axis" approximate impedance functions. A lumped parameter model consisting of two masses, two dampers and one spring was used as the physical model in this study. This model is called the Translational Head Injury Model (THIM) and was used earlier in the development of the Translational Energy Criteria for primate and human head impact analyses [1,2,3].

The approximate impedance functions were generated by adjusting the values of all the lumped parameters in the THIM model (K, C_1, C_2, M_1 and M_2) to obtain smooth variations between reference points (A-P, L-R, P-A or S-I direction). These approximate impedance curves at the "middle" point or 45 degree rotation from a reference point on the head surface were generated base on linear logarithmic variation. Next, the values of K, C_1, C_2, M_1 and M_2 for the reference and intermediate impedance curves were used to develop a quadratic function for estimating lumped parameter values of the THIM model at other impact locations on the head surface. Two independent sets of approximate functions were generated based on available experimental data. Finally, these two sets of mathematical equations were combined to produce a computer program which can be used to estimate impedance curves for any off-axis or indirect head impact.

The approximate impedance curves at 22½, 45, and 67½ degree rotations which were generated by the functions developed in this study show smooth logarithmic variations between the reference points. In general, the approximate impedance functions may be used to analyze off-axis head impacts which are much more common than direct head impacts in real world accident environments.

INTRODUCTION:

Statistically, some orientations of head impacts are more frequent and potentially more critical than other. Thus, greater attention should be focused on these important cases. But, it is still very desirable to be able to accurately predict and assess the injury consequences of head impacts at different locations. In the real world conditions, automobile related accidents are somewhat random in nature. As a result, the location of any head impact can not be mathematically predicted with much reliability prior to the accident itself. In addition, the head shape and structure are very complex. Thus, the dynamic responses of the human head will be quite different for the same impact force at different locations.

The objective of this research study is to estimate the head dynamic responses at various locations. Adjustments on the lumped parameters (K, C's and M's) were made to modify the impedance curves from the known reference values (A-P, L-R, P-A and S-I positions). Two independent sets of correction equations were generated assuming smooth variation between reference points on the head surface. These equations were combined to develop a set of modification functions which can be used to evaluate off-axis head impact dynamic responses.

BACKGROUND:

A. THE TRANSLATIONAL HEAD INJURY MODEL (THIM):

The THIM model is a one dimensional, three degree of freedom semi-definite lumped parameter model composed of two masses, one spring, and two dampers as shown in Figure 1. In this model, M_1 and M_2 represent the equivalent head mass, while K, C_1 and C_2 represent the effective stiffness and damping of the skull, brain and other head tissues/bones. General transfer function for the THIM model may be formulated from the equations of motion of the system. The driving point impedance function, F/V(s), may be expressed as,

$$F_1/V_1(s)=\frac{s\,[C_1M_1M_2s^2+(C_1C_2[M_1+M_2]+KM_1M_2)s+(K[M_1+M_2][C_1+C_2])]}{M_2C_1s^2+(C_1C_2+KM_2)s+K(C_1+C_2)}$$

In standard frequency response form, we may rewrite $F_1/V_1(s)$ as,

$$I=\left|\frac{F_1}{V_1}(i\omega)\right|=\frac{\omega\sqrt{[KM_1M_2+C_1C_2(M_1+M_2)]^2\omega^2+[K(M_1+M_2)(C_1+C_2)-\omega^2C_1M_1M_2]^2}}{\sqrt{[K(C_1+C_2)-\omega^2M_2C_1]^2+\omega^2(C_1C_2+KM_2)^2}}$$

and,

$$\angle\phi=\frac{\pi}{2}-Tan^{-1}\left[\frac{C_1C_2+KM_2}{K(C_1+C_2)-\omega^2M_2C_1}\right]+Tan^{-1}\left[\frac{C_1C_2(M_1+M_2)+KM_1M_2}{K(M_1+M_2)(C_1+C_2)-\omega^2C_1M_1M_2}\right]$$

where,

F_1 = Impact Force to the Head Model
K, C_1, C_2 = The THIM Model Spring and Dampers
M_1, M_2 = The THIM Model Masses
I = Magnitude of the Impedance Function at Frequency ω
ϕ = Phase Angle of the Impedance Function at Frequency ω

Variations of the parametric values of the lumped elements of the THIM model (K, C_1, C_2, M_1 and M_2) can result in modifications of the dynamic response characteristics of the overall system. Consequently, appropriate adjustments of model parameters can produce an expected shift in natural frequencies and/or damping ratios of the whole system.

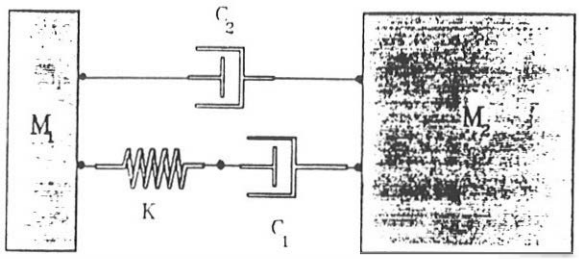


Figure 1: The Translational Head Injury Model (THIM)

B. THE INTERPRETATIONS OF THE THIM MODEL ELEMENTS

The parameters of the THIM model were perceived, in a general sense, to have physical representations. It appears from earlier studies that the THIM model elements have the following characteristics [5,6].

1. The summation of the masses, M_1 and M_2, equals to the total head mass.

2. Mass M_1 represents the portion of the skull/skin mass which is dynamically coupled with the rigid impactor.

3. The damper C_2 is constant for all directions and is believed to represent primarily the brain damping.

4. The stiffness K and the damper C_1, in series, form a complex skull stiffness system.

The first two characteristics of the THIM model elements listed above may be inferred directly from the experimental driving-point impedance curve of a cadaver head. At low frequency excitation, experimental data shows that the head acts very much like a rigid body as illustrated in Figure 2. In this case, the total head mass may be read off directly from the parallel mass line, M_1+M_2, on the impedance curve (on the left side of Figure 2). On the other hand, only a small portion of the head mass near the loading point will respond to high frequency excitation. The parameter M_1 of the THIM model represents this effective mass which is located beneath the impact point. Figure 2 also illustrates the high frequency effective mass, M_1, on the same impedance curve (right side).

The third characteristic of the THIM model elements may be explained by investigating two related experimental impedance curves of the same cadaver. These two curves represent the whole head and the skull only cases. The same cadaver was used in the experiment and analysis to maintain consistency of the material and structural properties. First, the mass for skull only model (M_1+M_2) was obtained by subtracting masses of brain and skin/tissue materials removed from the whole head structure. Then, the best THIM model was generated to approximate the frequency responses of the structure by adjusting the model lumped parameters (primarily K and C's) to fit the experimental curve for each case. Next, another THIM model was established to represent a skull plus brain model. This model was generated by (1) adding brain mass (1.5 kg) to the mass of the skull only model and (2) adjusting the values of damper C_1 and the spring K. In this process, the value of damper C_2 was kept unchanged at 350 N/(m/sec), the same value for the whole head model. The THIM model antiresonance frequency, ω_{nd}, is constrained to remain within the limits bounded by the two experimental impedance curves representing the whole head model and skull only model. The results of the model analyses are tabulated in Table I and illustrated in Figure 3. Comparison of the skull plus brain and

skull only model parameters in Table I indicates that the largest change of all model parameters is the value of the damper C_2 by more than 71%. This infers that the brain viscoelastic damping has significant influence on the value of damper C_2.

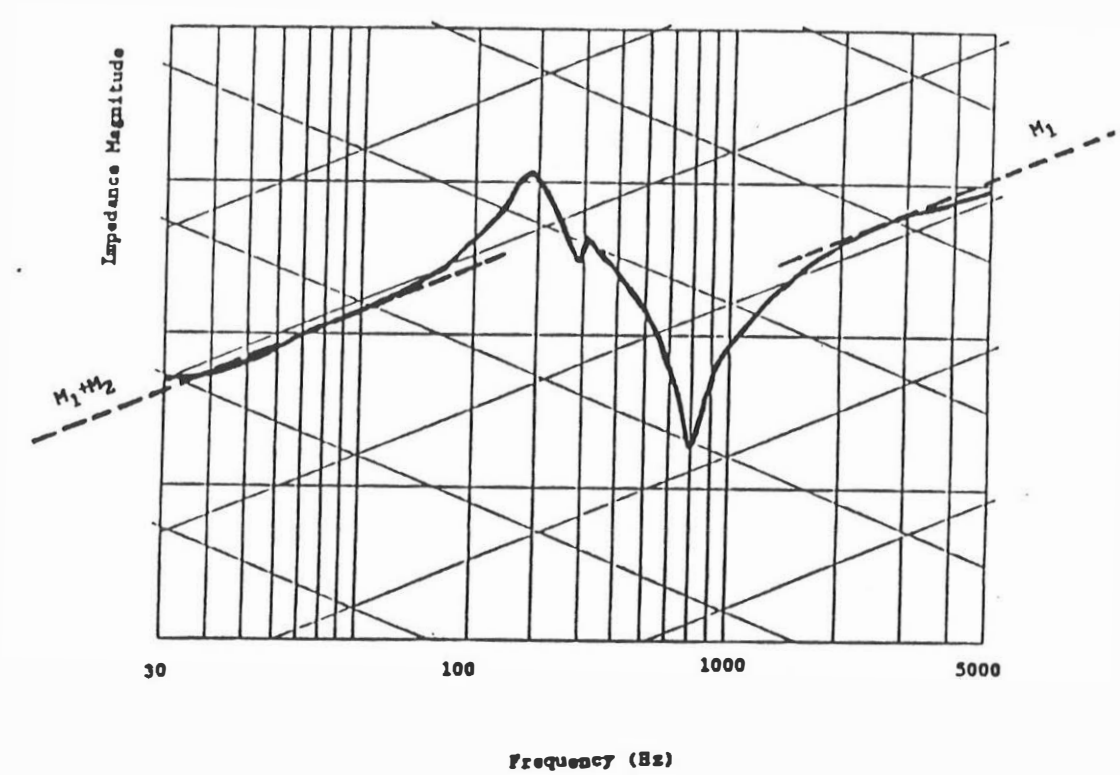


Figure 2: Cadaver Head Impedance Curve (A-P Direction)

Moreover, the brain material is homogeneous. Thus, it is expected to be independent of impact direction and location. From earlier studies of the THIM model, C_2 is estimated to be constant for all four impact directions [2,3]. This information further supports the conclusion made earlier about damper C_2.

Finally, the fourth characteristic of the THIM model elements may be explained by analyzing impact responses of the K-C_1 elements in serial combination as shown in Figure 4. Then, the predictions are compared with skull quasi-static stiffness data. From the equations of motion of the K-C_1 system, the dynamic responses of the overall system nonlinear stiffness may be expressed in frequency response form as follow,

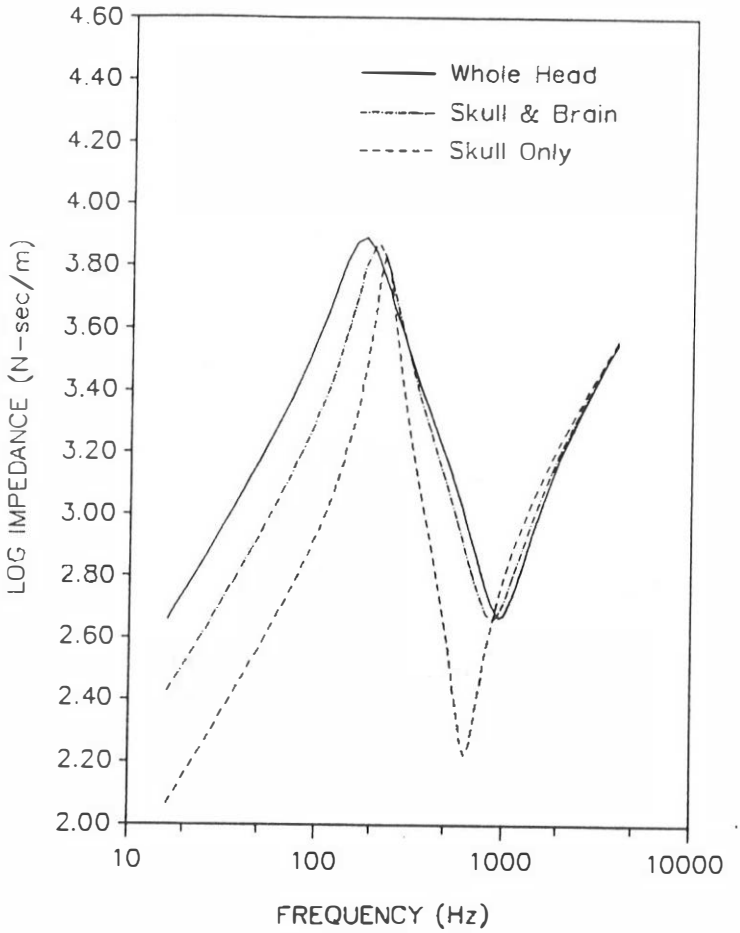


Figure 3: Comparison of Skull & Brain Effects on the Impedance Curves

$$K(\omega) = \left| \frac{F_1}{X_1}(i\omega) \right| = \frac{C_1\omega}{\sqrt{(C_1\omega/K)^2+1}}$$

Alternatively in time domain form with constant velocity input, the response may be expressed as,

$$F(t) = C_1 V_o \left[1 - \exp^{-K/C_1 \cdot t} \right]$$

where,

V_o = *Constant Velocity Input (m/sec)*

In order to compare dynamic and static stiffness, a multiplying factor of 5.7 was assumed and used in converting the experimental static stiffness corridors to dynamic stiffness corridors. Comparison of these experimental results to theoretical prediction of the K-C_1 system responses is shown in Figure 5 for the A-P impact direction. The good agreements observed here imply that K-C_1 in serial combination may be used to represent skull dynamic stiffness in selected cases.

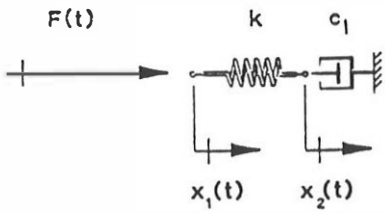


Figure 4: K-C_1 Combination Model

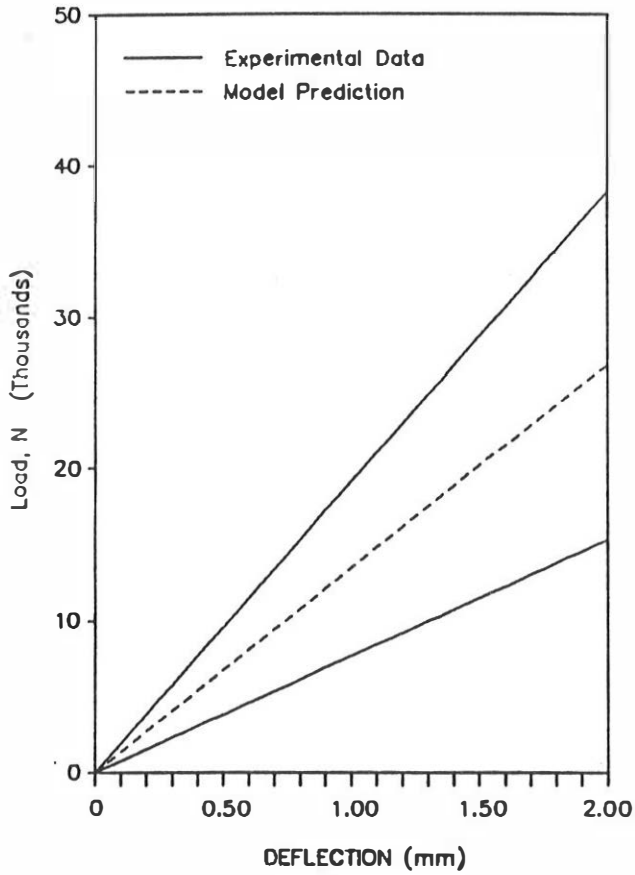


Figure 5: Skull Load-Deflection Curves and K-C_1 Model Prediction (A-P)

Table 1: Comparison of Skull and Brain Effects on THIM Model Parameters

	Whole Head	Skull & Brain[1]	Skull Only[2]	% Difference (1 & 2)
M_1 (kg)	0.15	0.15	0.15	0.0%
M_2 (kg)	4.39	2.50	1.00	-60.0%
C_1 (N/(m/sec))	$8.5 \cdot 10^3$	$9.5 \cdot 10^3$	$10.0 \cdot 10^3$	+5.3%
C_2 (N/(m/sec))	350.	350.	100.	-71.4%
K (N/m)	$5.0 \cdot 10^6$	$4.0 \cdot 10^6$	$2.0 \cdot 10^6$	-50.0%
ω^* (rad/sec)	1067	1265	1414	+11.8%

*: $\omega = (K/M_1)_{1/2}$, rad/sec

C. APPROXIMATE "MIDDLE" RANGE IMPEDANCE CURVE:

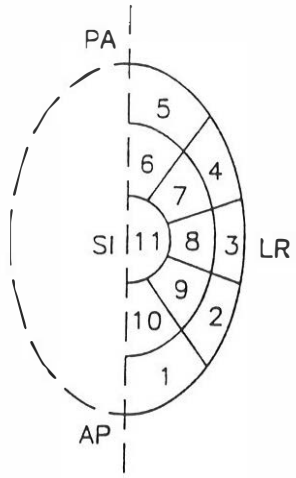
In this study, it is proposed that the "middle" range impedance curve represents the half-way point between two reference positions. This impedance will also be referred to as the 45° rotation position. Using this specification, we may define five independent "middle" approximate impedance functions denoted as APLR, LRPA, APSI, LRSI AND PASI. In addition, two other impedance functions may also be defined as APLRSI and PALRSI representing midpoints between three reference locations. The regions corresponding to these approximate impedances, with respect to the reference impact positions, are shown in Figure 6.

The values of the lumped parameters of the THIM model representing the approximate impedances were first estimated from the differences in the natural frequencies and the relative damping of the resonance and antiresonance. These rough estimates were later refined by trial and error until acceptable variations between reference points were obtained. The development of the estimation procedure is shown below.

From the parametric studies of the THIM model [3], the natural frequencies and damping ratios may be expressed as,

$$\omega_{nn}^2 = \frac{K(C_1+C_2)(M_1+M_2)}{C_1 M_1 M_2} \quad ; \quad \omega_{nd}^2 = \frac{K(C_1+C_2)}{C_1 M_2}$$

$$\zeta_n = \frac{1}{2} \frac{KM_1M_2 + C_1C_2(M_1+M_2)}{\sqrt{KC_1M_1M_2(C_1+C_2)(M_1+M_2)}}$$



	Description
1	AP
2	AP-LR
3	LR
4	PA-LR
5	PA .
6	PA-SI
7	PA-LR-SI
8	LR-SI
9	AP-LR-SI
10	AP-SI
11	SI

Figure 6: Mathematical Schematic of Head Surface

and,

$$\zeta_d = \frac{1}{2} \frac{KM_2 + C_1 C_2}{\sqrt{KC_1 M_2 (C_1 + C_2)}}$$

These equations are nonlinear and quite complicated to decouple. Fortunately, some simplifications are possible without much compromise on the accuracies of the analysis because of the large differences in the value of the variables in the THIM model. These assumptions are listed below.

$$(C_1 + C_2) \approx C_1 \quad ; \quad KM_2 + C_1 C_2 \approx KM_2$$

and,

$$\frac{(C_1 + C_2)}{C_1} \approx \frac{M_2}{\sqrt{M_2(M_1 + M_2)}} \approx \frac{M_1 + M_2}{\sqrt{M_2(M_1 + M_2)}} \approx 1.0$$

Next, substitutions of these expressions into natural frequency and damping ratio equations yield the following approximations,

$$\omega_{nn}^2 \approx \frac{K(M_1+M_2)}{M_1 M_2} \quad ; \quad \omega_{nd}^2 \approx \frac{K}{M_2}$$

and,

$$\zeta_n \approx \frac{C_2}{2\sqrt{KM_1}} + \frac{\sqrt{KM_1}}{2C_1} \quad ; \quad \zeta_d \approx \frac{1}{2C_1}\sqrt{KM_2}$$

Finally, we may approximate the parametric values of the THIM model at the desired resonance and damping from ω_n's and ζ's Furthermore, the same restrictions imposed on the THIM model were also enforced to assure consistency of the investigation process,

Restrictions:

$$C_2 = 157.6 \frac{N}{m/sec} = constant \quad ; \quad M_1+M_2 = 4.54 = constant$$

Approximation Equations:

$$K \approx \frac{4.54}{\left[\frac{1}{\omega_{nn}^2} + \frac{1}{\omega_{nd}^2}\right]}$$

$$C_1 \approx \frac{K}{2\zeta_d \cdot \omega_{nd}} \quad ; \quad M_1 \approx \frac{K}{\omega_{nn}^2} \quad ; \quad M_2 \approx \frac{K}{\omega_{nd}^2}$$

From these initial approximations, fine tunings of the parametric values were necessary to acquire the acceptable impedance responses. However, only minor adjustments were necessary in our derivations to attain the 45° approximate impedance curve.

DERIVATION OF THE APPROXIMATE IMPEDANCE FUNCTIONS:

From the previous section, we obtain the approximate impedance curves for the APLR, LRPA, APSI, LRSI, PASI APLRSI and PALRSI positions. The parametric constants for these off-axis models are summarized in Table II. And Figure 7 shows the variations of the APLR impedance function. These midpoint estimates represent some of the indirect impact orientations of the head. In the real world accident conditions, the exact location of the impact is most likely unknown or very difficult and sometime impossible to predict accurately. This essentially make the head orientation or impact location a variable in the analysis. Dividing the head into different segments or regions, as done earlier, will certainly help the investigation. However, it is now feasible to approximate head dynamic response at any impact location based on smooth logarithmic variation. This may be accomplished by performing quadratic curve fitting on the THIM model parameters utilizing the new 45° and corresponding reference parameters. The complete derivation procedure for the proposed approximate impedance functions is provided below in detail.

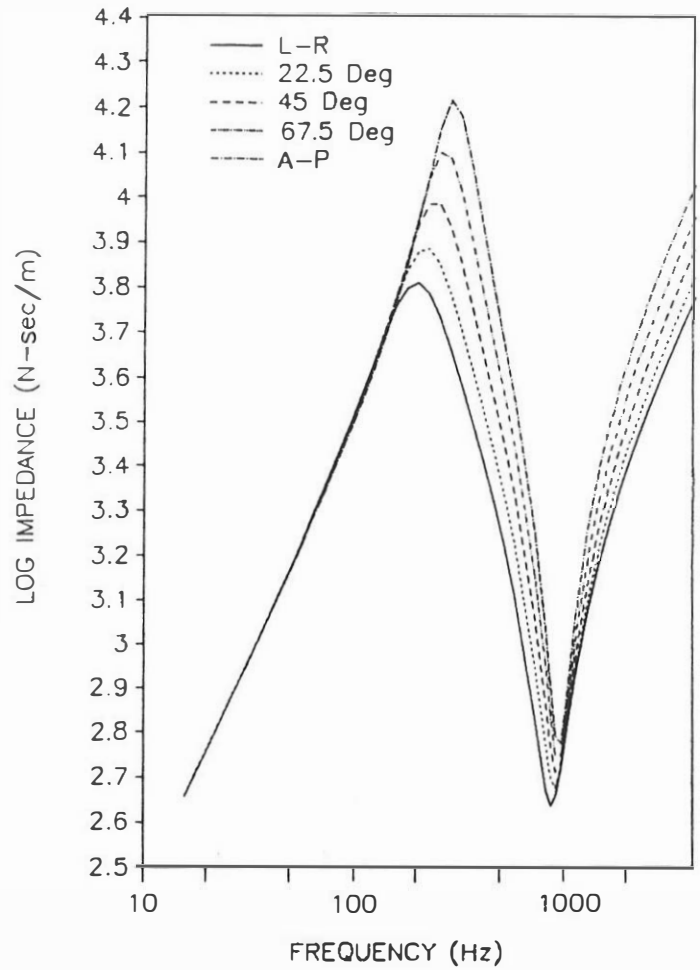


Figure 7: Variation of Approximate APLR Impedance Function

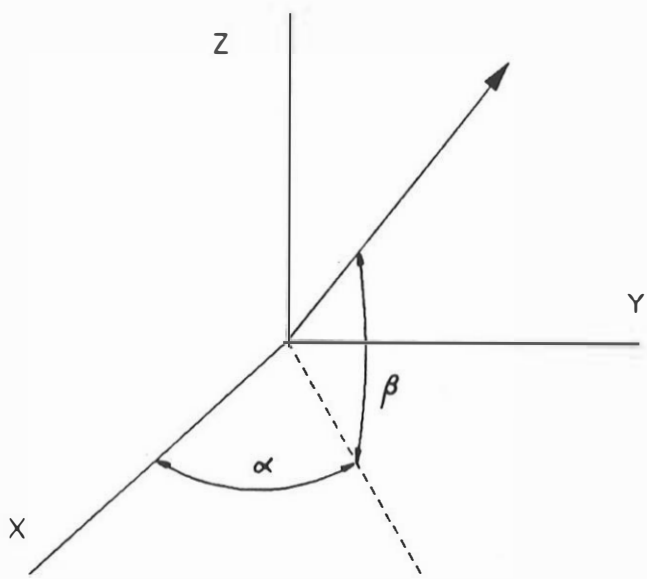


Figure 8: Head Orientation Angles

A general quadratic equation has the following form,

$$Y = A \cdot X^2 + B \cdot X + C$$

Table II: Approximate Off-Axis THIM Model Parameters

Impact Direction	K (N/m)	C_1 (N/(m/s))	C_2 (N/(m/s))	M_1 (kg)	M_2 (kg)
APLR	$9.6 \cdot 10^6$	$9.9 \cdot 10^3$	157	.31	4.23
LRPA	$6.9 \cdot 10^6$	$7.1 \cdot 10^3$	157	.25	4.29
APSI	$13.3 \cdot 10^6$	$20.5 \cdot 10^3$	157	.54	4.00
LRSI	$9.8 \cdot 10^6$	$12.5 \cdot 10^3$	157	.41	4.13
PASI	$10.0 \cdot 10^6$	$13.5 \cdot 10^3$	157	.43	4.11
APLRSI	$11.0 \cdot 10^6$	$16.0 \cdot 10^3$	157	.46	4.08
PALRSI	$9.8 \cdot 10^6$	$13.0 \cdot 10^3$	157	.42	4.12

In this development, two directional angles are required to locate impact position on a three dimensional surface such as the head. These directional angles are defines in Figure 8. Then, the THIM model parameters for the off-axis model are calculated based on the approximation using two sets of quadratic functions. The dependent variables in our analysis are three THIM model elements, K, C_1 and M_1. And the two distinctive independent variables are the orientation angles, α and β from the horizontal and vertical reference planes, respectively.

The first step of the derivation is to determine the impact directional angles from the components of the triaxial accelerometer measurement. Other equivalent and/or related measurements (impact forces, pressures, etc.) may also be used to calculate impact orientation. Two orientation angles, α and β, are calculated with respect to horizontal and vertical reference planes. Angle α represents the horizontal variation and has a range between 0° and 180° (not the complete 360° rotation because the head is assumed to be symmetrical along the sagittal or left/right plane). On the other hand, angle β represents the vertical variation and has a range between 0° to 90° measuring from the horizontal (Frankfurt) plane to the top of the head. The orientations are illustrated in Figure 8.

The next step in the development of the approximate functions is to solve for the quadratic equation constants, A and B. This may be accomplished in two computations. The first calculation is to estimate horizontal variation, follows by the second calculation which estimates vertical variation. In matrix form, the computations may be summarized as follow,

For APLR approximate function, we compute the spring constant, $K(\alpha)$, for the horizontal approximation,

$$K(0°) = 6.5 \cdot 10^6 \ N/m = K_o \quad \text{(L-R Direction)}$$
$$K(45°) = 9.6 \cdot 10^6 \ N/m$$
$$K(90°) = 13.5 \cdot 10^6 \ N/m \quad \text{(A-P Direction)}$$

and,

$$K(\alpha) = A_{K\alpha}\cdot\alpha^2 + B_{K\alpha}\cdot\alpha + K_o$$

which gives,

$$\begin{bmatrix} 2025 & 45 \\ 8100 & 90 \end{bmatrix} \begin{bmatrix} A_{K\alpha} \\ B_{K\alpha} \end{bmatrix} = \begin{bmatrix} 3.05\cdot10^6 \\ 7.00\cdot10^6 \end{bmatrix}$$

Solving for $A_{k\alpha}$ and $B_{k\alpha}$, we get

$$A_{K\alpha} = 2.22\cdot10^2\ N/m \quad ; \quad B_{K\alpha} = 5.78\cdot10^3\ N/m$$

In similar manner, the $C_1(\alpha)$ and $M_1(\alpha)$ computations yield,

$$A_{C_1\alpha} = 8.89\cdot10^{-1}\ N/(m/sec) \quad ; \quad B_{C_1\alpha} = 3.67\cdot10^1\ N/(m/sec)$$

and,

$$A_{M_1\alpha} = 1.23\cdot10^{-5}\ kg \quad ; \quad B_{M_1\alpha} = 1.11\cdot10^{-3}\ kg$$

The same procedure is then repeated for the vertical direction, β, using the results of the first series of calculations as a new initial conditions. Thus, the formulation becomes,

$$\begin{aligned} K(0°) &= K_{\alpha o} \\ K(45°) &= K_{\alpha 1} \\ K(90°) &= K_o = 13.0\cdot10^6\ N/m \quad (S\text{-}I\ Direction) \end{aligned}$$

and,

$$K(\alpha,\beta) = A_{K\alpha\beta}\cdot\beta^2 + B_{K\alpha\beta}\cdot\beta + K_o$$

which results in,

$$\begin{bmatrix} 2025 & 45 \\ 8100 & 90 \end{bmatrix} \begin{bmatrix} A_{K\alpha\beta} \\ B_{K\alpha\beta} \end{bmatrix} = \begin{bmatrix} K_{\alpha 1} - K_{\alpha o} \\ K_o - K_{\alpha o} \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

Next, solve for $A_{k\alpha\beta}$ and $B_{k\alpha\beta}$,

$$A_{K\alpha\beta} = \frac{D_2 - 2\cdot D1}{4050} \quad ; \quad B_{K\alpha\beta} = \frac{D_2 - 8100\cdot A_{K\alpha\beta}}{90}$$

Consequently, $C_1(\alpha,\beta)$ and $M_1(\alpha,\beta)$ for any impact position on the head surface may also be determined in the same manner (M_2 depends only on M_1 and C_2 is constant. Thus, only $K(\alpha,\beta)$, $C_1(\alpha,\beta)$ and $M_1(\alpha,\beta)$ functions are required in the analysis, not all five variables). A computer program is written to calculate these off-axis parameters. Variations of the approximation functions are shown in Figure 7 for the APLR region. Very good results are observed.

SUMMARY AND CONCLUSION:

In this paper, mathematical approximations of indirect head impact responses were proposed. The off-axis approximation functions assume smooth logarithmic variations of THIM model parameters between reference impact positions, which were measured experimentally. Seven intermediate impedance functions were generated to represent different impact regions of the head surface. Significant differences on the THIM model parameters were observed for each region. The computations of the approximate parameters, $K(\alpha,\beta)$, $C_1(\alpha,\beta)$ and $M_1(\alpha,\beta)$ involve two quadratic interpolations along horizontal and vertical reference planes. Very good results were obtained.

Experimental data is required to verify the prediction of the off-axis THIM model. However, the procedure suggested in this paper can be readily used to calculate a better set of model parameters when new data becomes available.

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