IMPROVEMENT OF THE PROTECTIVE EFFECTS OF MOTOR-
CYCLE HELMETS BASED ON A MATHEMATICAL STUDY

by

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1. Introduction

In the Federal Republic of Germany, 1,232 motorized two-wheel riders were killed in 1986, 22,510 suffered major and 46,142 minor injuries. The share of motorcycle riders in the overall number of traffic fatalities amounted to approx. 14 %. All riders of motorized two-wheelers are subject to the helmet wearing law. Law observance is very good (nearly 100 %).

Further reduction in the accident consequences for two-wheel riders requires the improvement of the passive accident protection components on a permanent basis. The helmet as the most important passive protection element is subject to the requirements as stated in ECE R 22.

The energy absorption effect of helmets is tested at an impact speed of 25 km/h (not including the secondary impact). Based on the required protection effect of the helmet, the possibilities of a mathematical analysis of energy absorption are described in this contribution. By varying the crash energy and helmet parameters such as shell thickness, density and thickness of padding, the resultant possibilities of system optimization are pointed out.

It is hoped that the results of this mathematical study will make it possible to substitute a load deflection test (a test e.g. in the speed range from 15 to 35 km/h) for the punctual test (test at an impact velocity of 25 km/h). This could then become part of ECE R 22.

2. Mathematical models

The objective here is the formulation of a mathematical model of an accident in its entirety. For the time being only the helmet is considered, that is to say leaving out the dynamics of the human body. Just the rigid mass of the head is coupled with a spring, e.g., the helmet. To check and support the results, two independent methods are used : (1) the transfer matrix method and (2) the finite element method.

2.1 Transfer matrix method

2.1.1 Helmet model

A diagram of the helmet model used in the transfer matrix method is represented in Fig. 2.1.1-1. The model is based on the following assumptions:

- rotational symmetry of the shell
- geometric linearity of the shell
- description of the energy absorbing padding only by its degree of freedom normal to the shell.

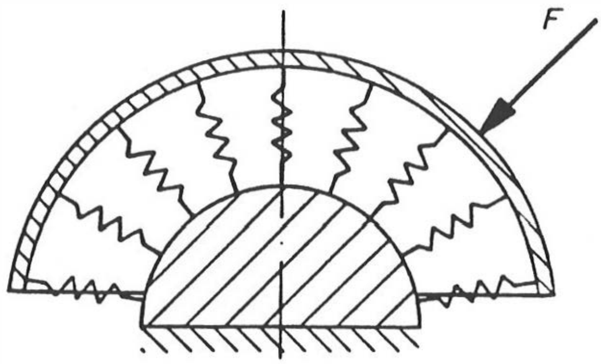

Fig. 2.1.1-1: Static helmet model

The static helmet model is used to determine the load deflection curve for the displacement in the direction of force application. The separation of the shell from the padding is taken into account.

If the helmet model as shown in Fig. 2.1.1-1 is reversed letting it crash at the chosen impact velocity, the "dynamic response" will be obtained by the consideration of a single degree of freedom system. An alternate system to describe the impact is thus given by a massless spring, which is the helmet, and the rigid mass of a head, including the influence of the body mass. The neglect of the mass of the helmet is equivalent to the assumption of coinciding static and dynamic deflection bending curve lines. This simplification can be explained by the large difference in the mass of the helmet and that of the head. It has to be kept in mind, however, that only a part of the helmet mass is "dynamically effective", the generalized mass.

2.1.2 Description of the shell by means of the thin shell theory

The thin shell theory on which this chapter is based has been derived in detail in /2/ and /3/. For a summary description, cf. /8, pp. 5-14/. The considerations below apply to rotation-symmetric shells, but extension to any shell form is possible. To begin with, let us establish a system of equations to describe a "ring element". A protective helmet can be discretized using several such ring elements (cf. Fig. 2.1.2-1).

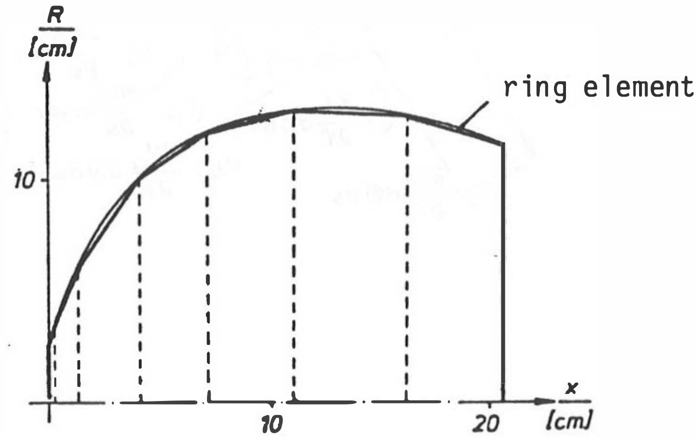

Fig. 2.1.2-1: Discretization of a protective helmet by means of ring elements

If the discretization is fine enough, the helmet can be approximated by means of cone shell elements. Its meridian will correspond to that of a standard helmet.

A cone shell element can be described by using a matrix differential equation. The basic element of the derivation of this matrix differential equation is the consideration of the equilibrium of forces and moments at the infinitesimal element (cf. Fig. 2.1.2-2). In part a of the Figure, the outer forces p_s, p_φ and p_z are represented as well as the inner forces. Part b represents the outer moments m_s and m_φ as well as the inner moments.

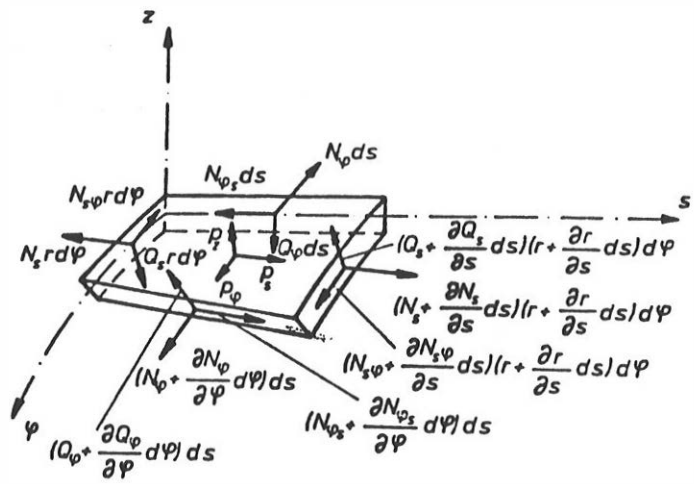

Fig. 2.1.2-2a

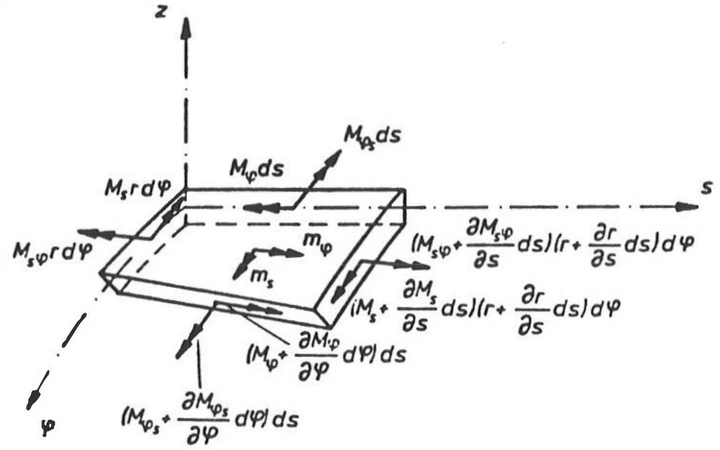

Fig. 2.1.2-2b

Figs. 2.1.2-2a and 2.1.2-2b: Equilibrium at the infinitesimal element

The displacements in the direction of the coordinates s, φ and z are denoted by u, v and w respectively. Using the Fourier series approach, the dependence of these values (which describe the behavior of the element) on the circumferential direction can be removed. This results in an ordinary nonhomogeneous matrix differential equation of the form

$$\frac{\partial}{\partial s} \underline{z} = \underline{A}(s)\, \underline{z} + \underline{l}(s)$$

with an eight by eight differential matrix $\underline{A}$ (s) and the state vector

$$\underline{z} = (v, u, N_s, N^*_{s\varphi}, w, w', M_s, Q^*_s)^T.$$

The differential matrix is not shown as it is beyond the scope of this paper. The bedding of the shell on the padding is introduced as deformation-dependent load (product of stiffness k and displacement w). Due to the development of the Fourier series in the circumferential direction, the differential matrix represents a function of the "Fourier wave numbers" (circumferential wave numbers m). The values

$$N_s, M_s, Q^*_s, u, w, p_s, p_z, m_s$$

vary in the circumferential direction with $\cos(m\varphi)$ and the values

$$N^*_{s\varphi}, v, p_\varphi, m_\varphi$$

with $\sin(m\varphi)$.

The linearity of the differential equation and the orthogonality of the trigonometric functions enable separate consideration of each individual circumferential wave m and arrival at the total solution by summing up the partial solutions.

The system of the differential equations of a cone shell element is integrated by means of the Runge-Kutta-Fehlberg method. The resulting integral or transfer matrices $\underline{U}_i^{i+1}$ describing the transfer of a state vector by a cone shell element are arranged as follows in an overall system of equations:

$$\begin{bmatrix} \underline{U}_1^{2*} & -\underline{I} & & & & & \\ & \underline{S}_2^3 & -\underline{I} & & & & \\ & & \underline{U}_3^4 & -\underline{I} & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & \underline{S}_{n-2}^{n-1} & -\underline{I} & \\ & & & & & \underline{U}_{n-1}^{n} & -\underline{I}^* \end{bmatrix} \begin{bmatrix} \underline{z}_1^* \\ \underline{z}_2 \\ \underline{z}_3 \\ \cdot \\ \cdot \\ \underline{z}_{n-1} \\ \underline{z}_n^* \end{bmatrix} = \underline{1}$$

The matrices $\underline{S}_i^{i+1}$ are in fact ring frame matrices, cf. /11, p. 40/, /9/. In the case of a helmet, these ring frame matrices "degenerate" into transformation matrices. They help transform the coordinates between the local systems of coordinates of the cone shell elements. To save computation time, they also include the padding effect. Because of this, a change in the padding stiffness does not require renewed calculation of the transfer matrices. The integration of the padding stiffness is thus carried out separately and not by the integration of the differential matrix. The sign + implies that the matrices and vectors in question are reduced by the corresponding columns and components due to the introduction of boundary conditions. Vector $\underline{l}$ consists of the load vectors of the transfer matrices. In addition, it also contains "pseudoforces" (cf. Chapter 2.1.3) describing the deviation of material behavior from linearity. The matrix system is 136 by 136 in size. The system of equations describes helmet behavior for one circumferential wave only. The loading case in question, e.g., Fourier's analysis of load function, determines the number of circumferential waves to be taken into account.

2.1.3 Numerical approach to consider nonlinear padding behavior

Fig. 2.1.3-1 shows the compressive stress-strain diagram for polystyrene in which density is used as a parameter, cf. /4, p. 103/. Maximum strains are achieved within 5 - 10 ms corresponding to a strain velocity of 100 - 200 s^{-1}. This is comparable to the time period of 2 - 4 ms in which the force applied in a helmet test attains its maximum. The curves describe the material properties only in the direction of impact. Towards the end of the range of linearity, the polystyrene cells begin to fracture. Finally, when all cells have fractured, the material "becomes a hindrance to itself". Caused by the structural destruction, a smaller amount of compressive stress will result when loading is repeated.

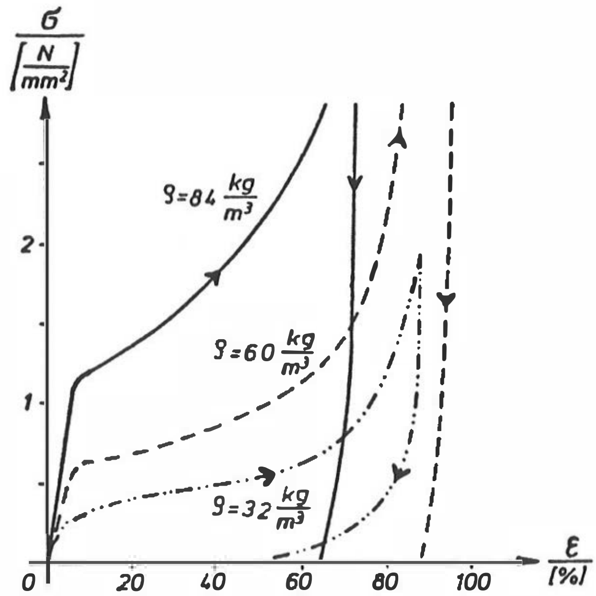

Fig. 2.1.3-1: Compressive stress-strain diagram for polystyrene /4, p. 103/

The nonlinear behavior of the protective padding is taken into account by means of an incremental equilibrium process. The external load is changed in increments. The calculation within individual load increments is carried out according to a linear law. In so doing, "pseudoforces" are applied in an iterative process to accomplish a correction with respect to the material criteria (cf. Fig. 2.1.3.-2).

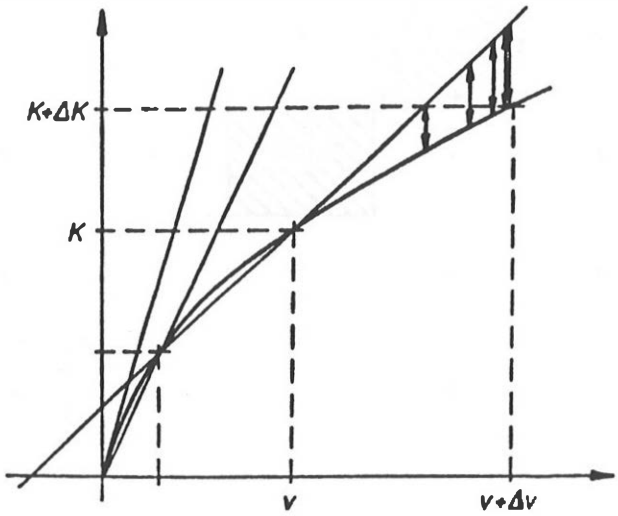

Fig. 2.1.3-2: Iteration by means of pseudoforces

Each iterative step starts at point (v, K). The helmet is subjected to the incremental load and the pseudoforce distribution, respectively, until point (v+Δv, K+ΔK) has been approximated. The whole procedure is repeated until the load deflection curve is determined point by point. To ensure convergence, two measures have been provided in the computer program. Firstly, following the Broyden-Fletscher-Goldfarb-Shanno method /1, p. 539/, the linear law is continously adjusted, depending on the location and as determined by the secant. Secondly, and also for reasons of accelerating convergence, the Aitken δ^2-method is applied /6, p. 275/. The separation of the shell from the padding is taken into account.

The padding stiffness k depends on the state of deformation of the helmet and is a function of the circumferential coordinate φ. Constant k in the differential matrix of the cone shell element has to be substituted by a Fourier series. The product of two Fourier series is transformed into a trigonometric series by means of the addition theorem

$$2\cos(i\varphi)\cos(j\varphi) = \cos((i-j)\varphi) + \cos((i+j)\varphi)$$

This leads to the coupling of the circumferential waves. An illustration of the coupling is given by the basic diagram in Fig. 2.1.3-3.

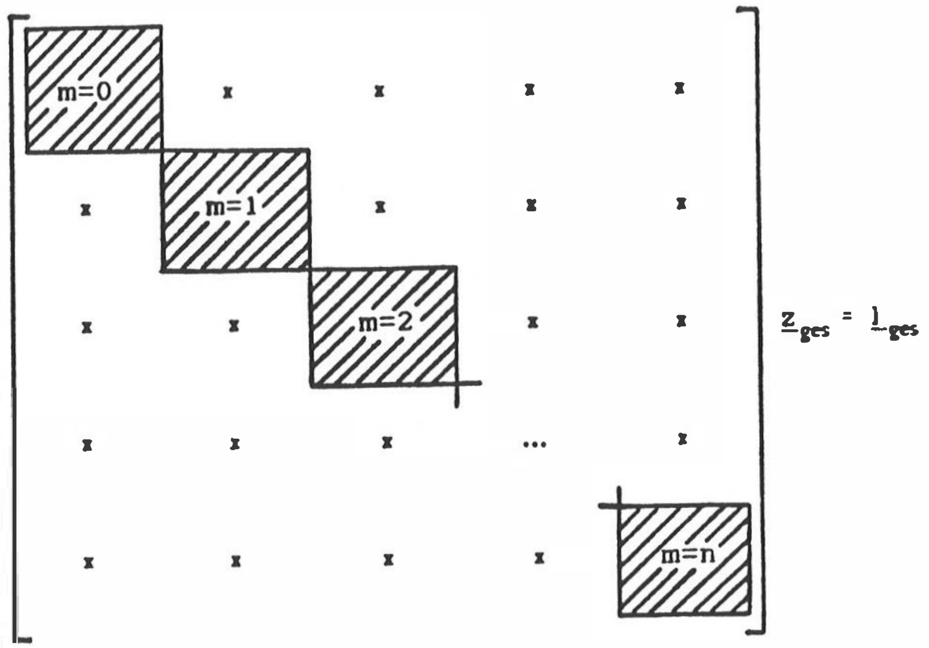

Fig. 2.1.3-3: Basic diagram of the coupling of circumferential waves by means of the padding (coupling terms x)

The coupling changes at each point of equilibrium of the load deflection curve. Each computed state of helmet deformation is associated with a corresponding distribution of pseudoforces. The Fourier coefficients of the distribution of pseudoforces are added at the next step of computation $\underline{l}_{ges}$. Details are given in /10/ and /9/.

2.1.4 Results

The determination of the load deflection curves in respect of an impact at the crown has been carried out in /12/ where the procedure is thoroughly documented. The loading case is illustrated in Fig. 2.1.4-1.

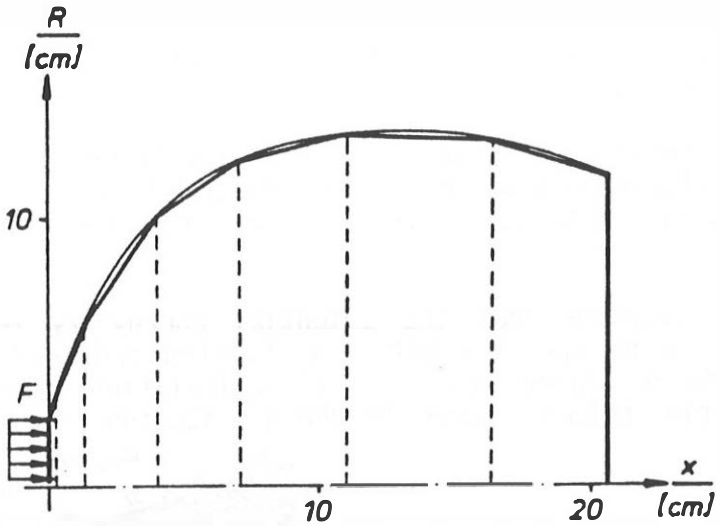

Fig. 2.1.4.-1: Computation model for impacts at the crown

As an example, a load deflection curve for a given load applied to a stiff shell (6 mm thick) is represented in Fig. 2.1.4-2. The load deflection curve characteristic tends to become linear when shells are less thick.

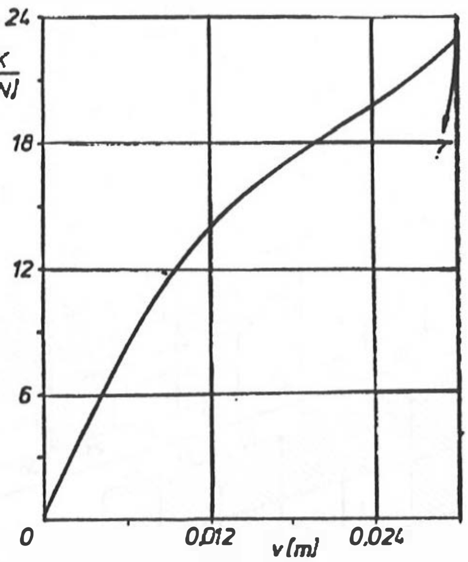

Fig. 2.1.4-2: Load deflection curve for a motorcycle helmet (impact at the crown)

The curve representing the state when the load has been removed can only be calculated after the completion of the current tests on polystyrene. Energy dissipation in the case of polycarbonate-polystyrene helmets generally amounts to 70 % /13, p. 28/. From the force equilibrium at maximum load, which is a function of the impact energy, we obtain the maximum head accelerations, expressed as the multiple of the gravitational acceleration of a head mass of 5.6 kg (ECE R 22/02, head circumference of 60 cm) and summarized in Fig. 2.1.4-3. A total of 27 helmet variants has been studied with shell thicknesses of 2, 3 and 4 mm; padding thicknesses of 25, 30, and

35 mm; and padding densities of 32, 60, and 84 kg/m³. The studies resulted in the following effects:

- increase in acceleration with an increase in the thickness of the shell
- increase in acceleration with an increase in the density of the padding
- minor reduction in acceleration with an increase in the thickness of the padding.

The shaded areas indicate that the potential energy is less than the kinetic energy. As a result, the helmet is blocked and head acceleration moves towards limit ∞. Based on this static calculation there results the possibility of testing helmets using the 300 g criterion required under ECE R 22/02.

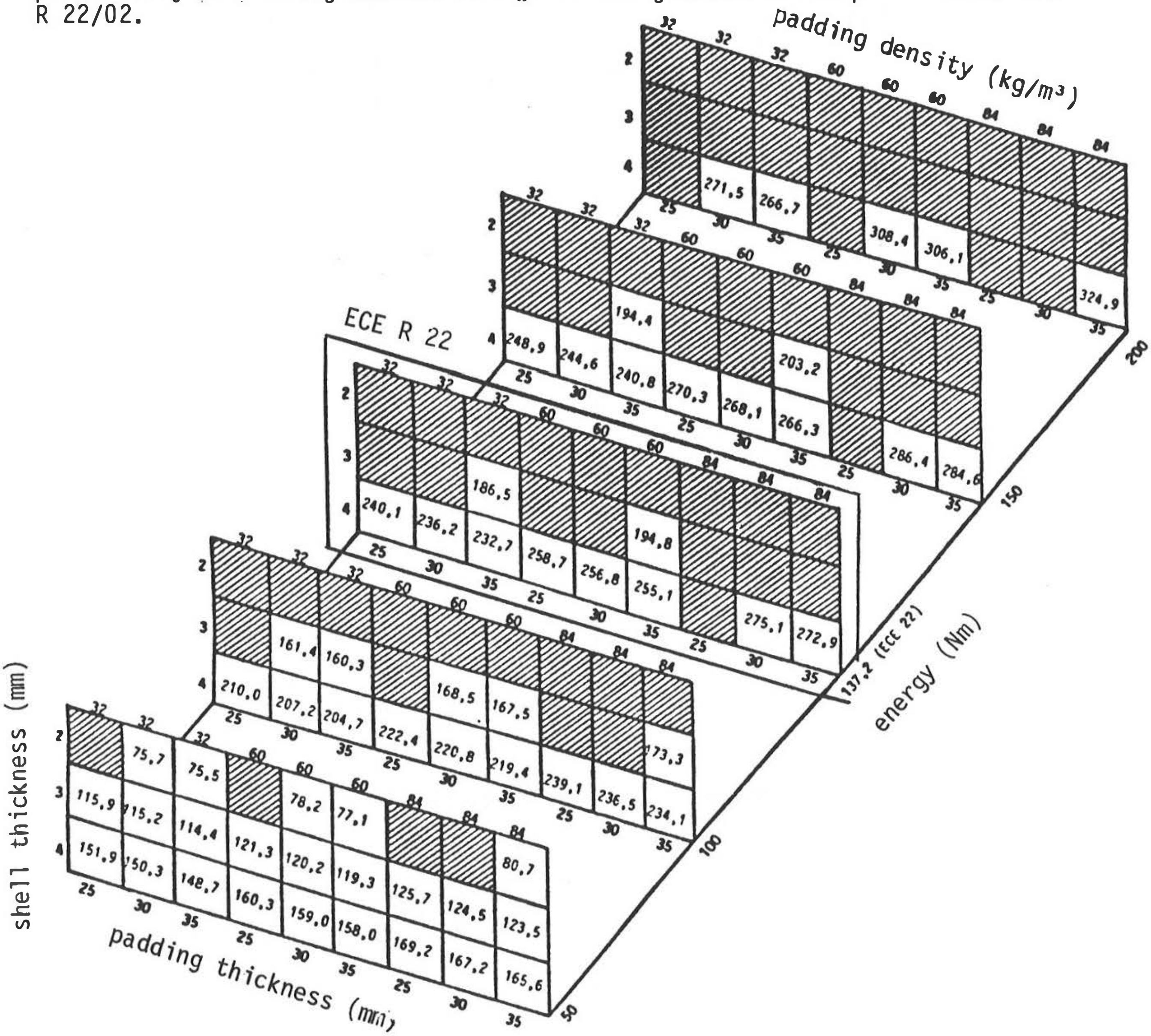

Fig. 2.1.4-3: Maximum head accelerations as the multiple of gravitational acceleration resulting from impacts at the crown in the case of a head mass of 5.6 kg (polycarbonate-polystyrene helmets)

Provided sufficient energy absorption capacity is ensured, the best helmet variant out of the polycarbonate-polystyrene combinations proved to be one of relatively small shell thickness, small padding density and, at low effect, thick padding. This applies to a helmet of a shell thickness of 3 mm, padding density of 32 kg/m³ and padding thickness of 35 mm (according to ECE R 22/02 at 137.2 Nm, corresponding to an impact velocity of 7 m/s).

All helmets studied show a similar relationship between head acceleration and energy since the calculations were based on only one type of helmet structure. In Fig. 2.1.4-4, a comparison of this functional relationship is shown for the calculated best helmet and the helmet rated "ideal" in a consideration of physical limits.

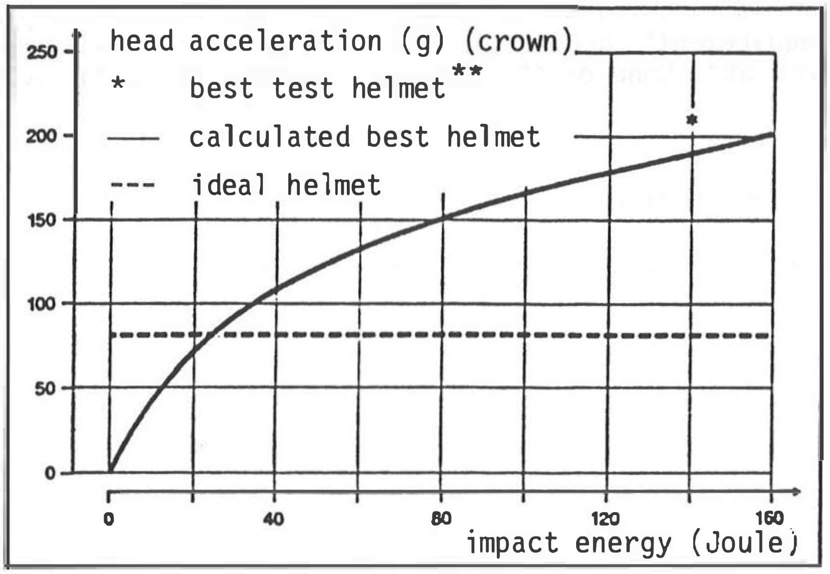

Fig. 2.1.4-4: Maximum head acceleration as a function of impact energy

Both functions are based on an inert mass of 5.6 kg. For the ideal helmet, a deformation path of 30 mm at 137.2 Nm has been assumed. At maximum energy (160 Nm at 7.6 m/s), the resulting deformation of the shell and padding is 35 mm. At the energy of 137.2 Nm, the curves differ by about 103 g which is large enough to be taken as an indication of the likely success of further development efforts. The diagram also includes the helmet which came off best in a test series.

Due to the lack of data on helmet structure and mass of test headform, the comparison of test and calculation results is difficult and possible in certain areas only. Test accelerations ranging from 209 g to 278 g** have to be compared with results of calculations ranging from 187 g to 275 g (rounded off).

** "Motorrad", 11/1987, pp. 110-136

With respect to the calculation results, it has to be pointed out that the effects not considered--contact problem with the level-surfaced rigid wall in impacts (according to ECE), three dimensional properties of the padding, axi-symmetrical deviation and geometric nonlinearity of the shell--either proved to be of minor importance if considered separately or to at least in part compensate for one another. The computation program can take the contact problem into account. It has been neglected here because the relatively rough discretization of the helmet did not enable a tight overall fit of the helmet structure.

To check the helmets in the parameter study with respect to the 150 g criterion of ECE R 22/02, displacement, velocity and acceleration were calculated as functions of time, cf. /8, pp. 233f./. Regarding the curves resulting when the loading is removed (hysteresis), assumptions which according to the ECE are fairly safe were necessary. All helmets satisfying the "static requirement" under the ECE regulation are also meeting the "dynamic requirement". None of the helmets exceeded 150 g for longer than 4 to 4.5 ms.

2.2. Finite element results

The finite element calculations were carried out by means of the ABAQUS programm /5/. A total of three finite element networks are available, corresponding to the impact points as specified under ECE R 22/02 (Fig. 2.2-1 to 2.2-3).

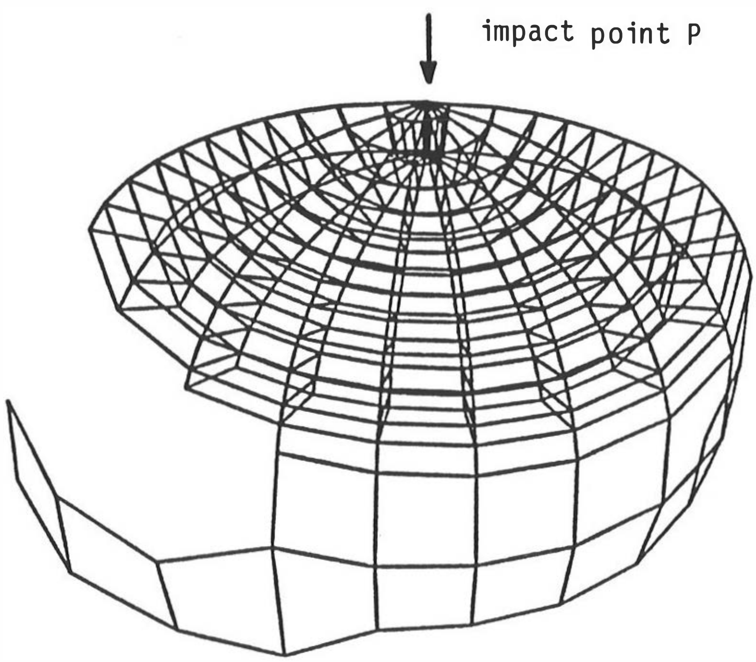

Fig. 2.2.-1: Finite element network for impacts at the crown (impact point P according to ECE)

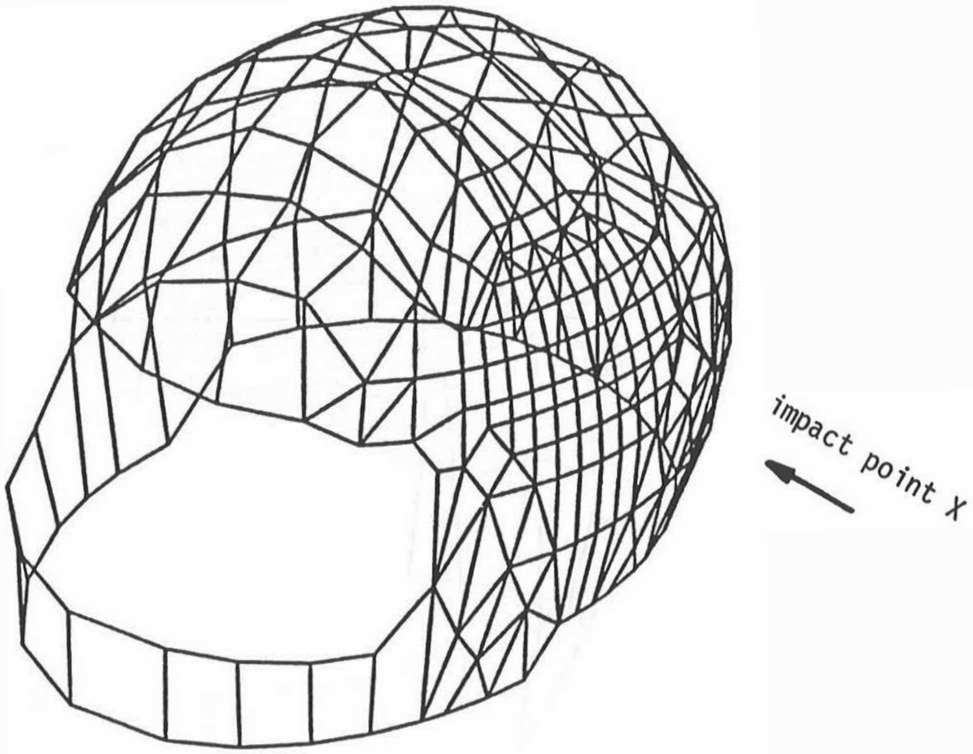

Fig. 2.2-2a: Helmet shell alone

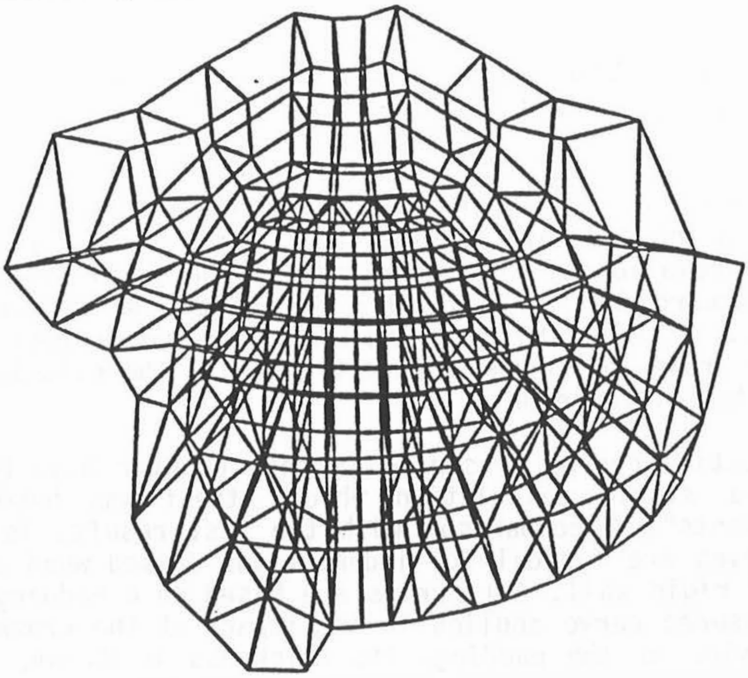

Fig. 2.2-2b: Protective padding alone

Fig. 2.2-2a and 2.2-2b: Finite element network for lateral impact (impact point X according to ECE)

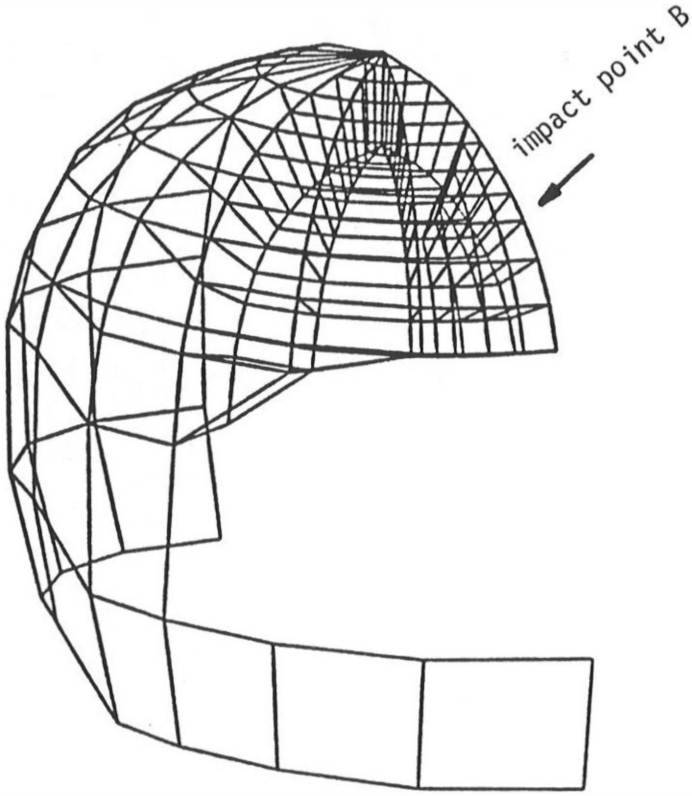

Fig. 2.2-3: Finite element network for impacts at the forehead (impact point B according to ECE)

The shell consists of shell elements and the padding of volume elements. The padding was modeled only inside the impact area (projection area) enabling the simulation of the separation of the shell from the padding in other areas. According to /7, p. 9/2 /, plastic deformation takes place within an area of 80 - 100 mm diameter. The symmetrical properties of impacts at the crown and forehead permit limiting the calculations of these loading cases to half a helmet.

The load deflection curves presented in this Chapter have been determined by means of a static calculation which itself was based on "dynamic material constants". A comparison with the test results is shown in Fig. 2.2-4. The curves are typical for indentations caused when crashing into a level-surfaced rigid wall. All curves are based on a padding density of 32 kg/m³. The measured curve applies to the impact at the crown and describes only the behavior of the padding. Its thickness is 23 mm. The calculated curves apply to the impact at the forehead and are based on the assumption of isotropic padding behavior. Owing to the relatively small projection area, the effect of the isotropic nature of the padding--still not confirmed by material tests--in impacts at the forehead has been minimal. Two variants of boundary conditions were introduced to take the slipping of the padding on the hair in certain places into account. A calculation thickness of padding and shell of 30 mm and 4 mm, respectively, was used.

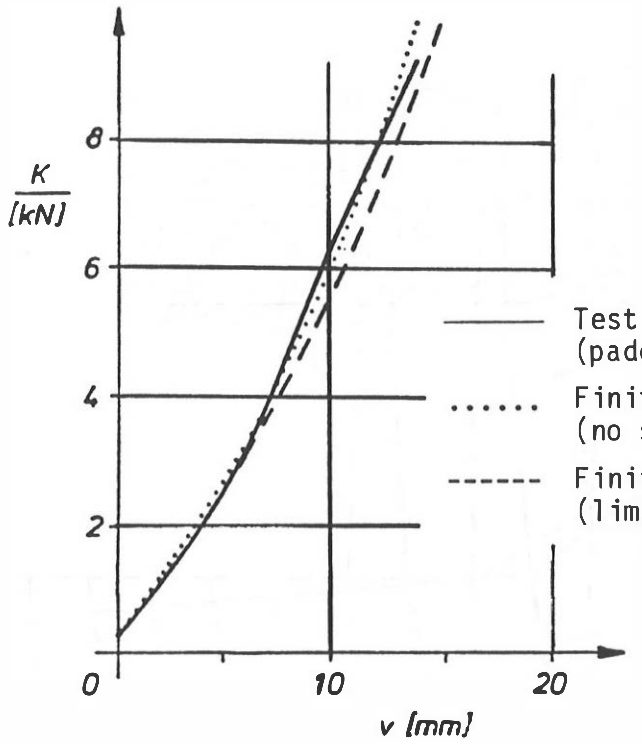

Fig. 2.2-4: Test versus calculation (level-surfaced rigid wall)

The contour of a deformed helmet is shown in Figs. 2.2-5a and 2.2-5b. For the load deflection curve of boundary condition b, the time to attain maximum head acceleration according the ECE dropping test is 5 ms. It has been determined by calculating a single degree of freedom system.

The load deflection curve as a function of shell thickness, padding density and thickness in a case of lateral loading is represented in Fig. 2.2-6. Here the same trends are observed as in the transfer matrix calculations. Force has been applied by means of element loads.

A direct comparison of measurement and calculation for an impact at the crown is given by Fig. 2.2-7. The measurement points are taken out of /13; Beilage 17, Bild 21, Helm B/. The boundary conditions of measurement and calculation are:

	Measurement	Calculation
Shell thickness	3,9 - 4,2 mm	4 mm
Padding thickness	28 mm	30 mm
Padding density	57 $\frac{kg}{m^3}$	60 $\frac{kg}{m^3}$

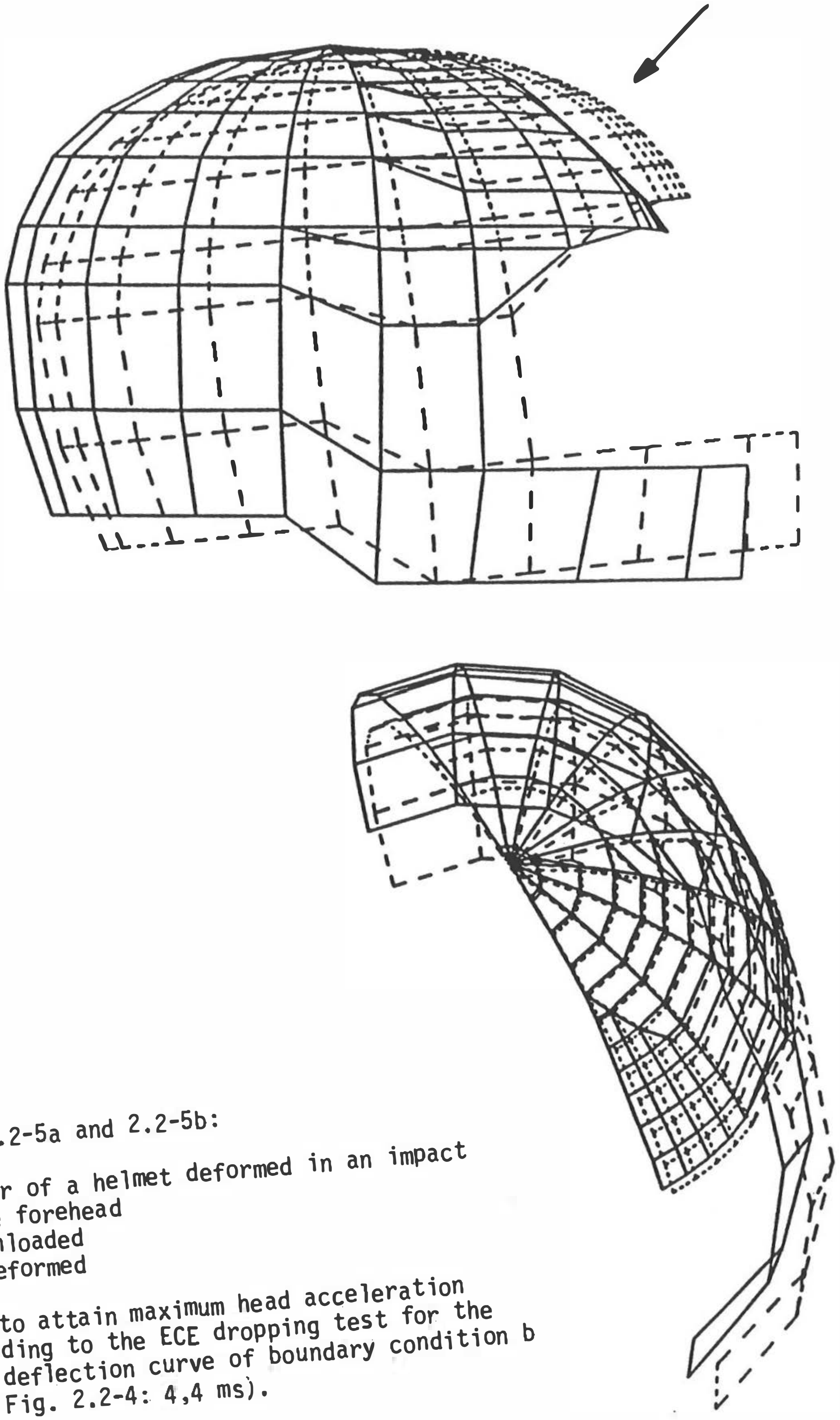

Fig. 2.2-5a and 2.2-5b:

Contour of a helmet deformed in an impact at the forehead
--- unloaded
___ deformed

Time to attain maximum head acceleration according to the ECE dropping test for the load deflection curve of boundary condition b (cf. Fig. 2.2-4: 4,4 ms).

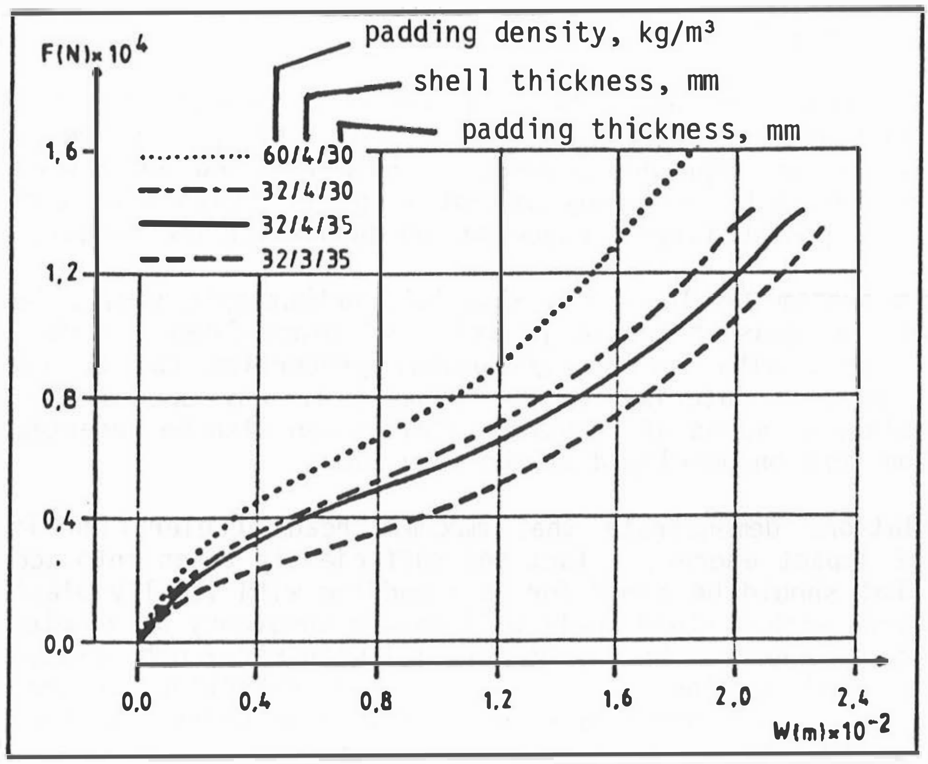

Fig. 2.2-6: Load deflection curve as a function of shell thickness, padding density and thickness (side impact, finite elements)

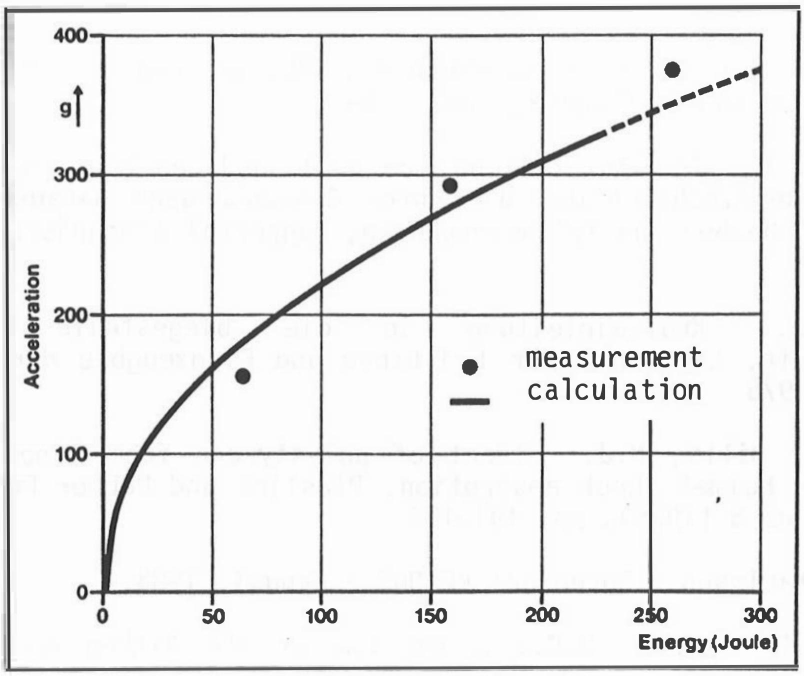

Fig. 2.2-7: Comparison of measurement and calculation for an impact at the crown

3. Possible application of research findings

Based on previous research, it is possible to undertake relatively quick advance calculations of the properties of any protective helmet. The development of an improved protective helmet based on measurements and calculations would be advantageous not only for reasons of costs but also because of the possibility of checking and supporting the results.

The program system developed can also take orthotropic shells into account. Apart from the quasiisotropic plastic and glass-fiber reinforced plastic helmets, helmets with various orthogonal properties can be calculated as well. The padding material is not subject to any special criteria. The parameter study by means of transfer matrices can also be undertaken by using a PC program version developed at the same time.

The calculations demonstrate that maximum head acceleration is greatly a function of impact energy, a fact not sufficiently taken into account in ECE R 22/02. What should be aimed for is a padding with ideally plastic behavior which combined with a rigid shell will ensure constancy of acceleration over a wide range of energy. The use of material with honeycomb structure between shell and comfort-padding may be a step in this direction. In contrast to expanded plastics, this might produce a favorable degressive behavior in the range of large deformations. Even the combination of a rigid shell with the polystyrene material used on the building sector results in noticeable mathematical reductions in head acceleration.

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