DYNAMIC SIMULATION OF THE CERVICAL SPINE WITH USE OF THE DIFFERENTIAL DISPLACEMENT MATRIX

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ABSTRACT

An elasto-dynamic model of the human spine which includes an accurate three-dimensional geometry of vertebra together with ligaments and muscles which are biomechanically represented by non-linear springs and dampers is developed.

With use of the Differential Displacement Matrix the model leads to non-linear differential equations which are then solved for the continuous simulation. A detailed analysis of the modelling and computation is carried out on the cervical spine including 126 cervical ligaments and eight spinal disc elements. In the computer program a subroutine is included such that each disc performs as the real available experimental data of the disc. Actual computation is carried out by dual CDC 6400 and then graphically represented by an interactive graphic terminal. This is to develop and advance an accurate model along with the computational techniques of the spinal simulation for a detailed and localized study of process of injury on the spine under impact situation.

The results so far are very encouraging mainly due to success in the use of the differential displacement matrix to the better formulation of the differential equations and success in the simulation with continuous numerical convergencies obtained.

1. INTRODUCTION

Doctors of Chiropractic dealing with a cervical spinal manipulative adjustment must be able to evaluate the effectiveness of the adjustment or develop new adjustment techniques through the use of dynamic analysis systematically. Designers of systems that interact with the human body must be able to evaluate safety devices to insure the safety of those humans. Automobiles, mass transportation, pilot ejection seats, and sporting goods are just a few examples. The neck and head are a relatively fragile, yet critical, part of the body that must be protected. Dynamic analysis of the skull and cervical spine is essential in these evaluations.

Since humans are the subject, actual experimentation and testing is not possible, as much of the testing would be of a destructive nature. Obviously, a technique for dynamic simulation of the cervical spine is needed. Then the doctor or designer could optimize the performance of his system. The use of cadavers for testing has several disadvantages. Among them are the lack of availability, changes in the physical properties which occur quickly after death, need for transducers to obtain the data, and the poor repeatability of such tests. A mechanical model could conceivably be constructed to simulate the human spine, but it, too, would have disadvantages. It would have to be complex (and thus expensive); unless a standard model were developed, repeatable data would be difficult to obtain; transducers would be needed to interpret physical motion into usable data; values for the parameters such as damping property of a ligament would have to be obtained from the body; and simple parameter changes would require an expensive and time consuming mechanical and material change.

Mathematical modeling of the cervical spine is possible using the digital computer. A computer simulation has several advantages: once the program is developed, it is simple to use and to obtain repeatable data; the parameters in the simulation can be changed quite easily, enabling many different cases to be analyzed; and the mathematical model could be more accurate than a physical model, since no transducers are required to quantisize the simulation into interpretable data. An efficient computer model of the cervical spine is the best way to approach this dynamic analysis problem.

2. MODELING THE ANATOMICAL FEATURES OF THE CERVICAL SPINE

The first step in the development of a discrete element model for the cervical spine is to determine how the actual anatomical features can best be represented in the computer model. The body parts that need to be modeled are the skull, the seven cervical vertebrae, the intervertebral discs, the connecting ligaments, the muscles attached to the skull and vertebrae, and the joints and articulating facets between the bony tissue. Once model elements are established for these parts, a method for writing and solving the equilibrium equations must be developed.

Gross Anatomy of the Spine

The cervical spine consists of seven vertebra interconnected with intervertebral discs, ligaments, and muscles and contact each other through articulating facets. The joint between the first (atlas) and second (axis) vertebrae does not have a disc, but is basically a pivot joint allowing rotation, flexion, and extension. The skull and atlas are connected with a hinge joint allowing only flexion and extension. The articulating facets between other vertebrae are basically sliding joints. The cervical spine is connected to the thorasic spine through a connection similar to the connections between the axis through C7 vertebrae. Muscles also connect the cervical spine to the rest of the body to control movement.

Vertebrae and Skull

The vertebrae and skull are composed of bone tissue and are relatively rigid compared to the other body parts. They do flex to some degree under loads, but this flexure contributes very little to the behavior of the system. Under great acceleration, the center of mass of the skull will shift somewhat due to movement of the brain, but that shift is ignored in this model. The skull and vertebrae are modeled as rigid bodies that have mass and transmit forces and moments.

Ligaments

Ligaments are composed mainly of bundles of collagenous fibers placed parallel with, or closely interlaced with, one another, and have a white, shining, silvery appearance. They are flexible, but strong, tough, and inextensible. Some ligaments consist entirely of yellow elastic tissue, as the ligamenta flava. The elasticity serves as a substitute for muscle power. In this model, the following assumptions are made about the ligaments: 1) a bundle of ligament fibers is modeled by one or more massless nonlinear spring-damper parts; 2) a ligament is connected at a point; and 3) no moments are generated at the connecting points as the ligaments are assumed to be perfectly flexible.

Intervertebral Discs

Intervertebral discs are very complex to model. Ligaments are basically tension devices and can easily be modeled by an element like a spring. The discs, however, attach over a large area and carry loads in all possible ways, and thus cannot be replaced by a few simple elements. Instead, the discs are modeled using the experimental data directly. Data is collected by first establishing a reference frame on each of two adjacent vertebrae at their geometric centers. One vertebra is fixed while the other is displaced in the desired way and force readings are taken. The positive and negative directions of the six degrees of freedom of the moving vertebra provide twelve different movements that must be tested. However, the lateral symmetry of the vertebrae eliminates three of these movements, as the positive and negative directions of those three should be the same. This symmetry leaves nine movements to be tested, namely: flexion, extension, rotation, lateral flexion, a transversal plane motion along the coronal direction, a positive and negative transversal plane motion along the midsagittal direction, elongation, and compression.

The data for each motion in each vertebral pair is then put into equation form through a curve fitting technique where

$$Y = f(X)$$

$$\text{Such as } Y = aX + bX^2 ,$$

Y is the moment or force, X is the displacement (rotation or translation), and a and b are determined by the curve filling. This testing of actual vertebral pairs must also include the dynamic properties of the intervertebral disc to establish the damping properties of the disc. This information about the curves is then written into a subroutine which will calculate the forces and moments exerted by the disc given the relative displacement and velocity of two adjacent vertebrae. It is assumed that the discs are massless. Since the relative displacements between adjacent vertebrae will be small, every displacement in a complex displacement is assumed to be independent and the overall effect of a complex displacement is the sum of each independent displacement.

Muscles

Muscles work in two modes. They can exert force by contraction, or they can be passive and function similar to an elastic ligament. They are connected by flexible tendons. In the passive mode, the muscle is modeled exactly like a ligament with the same assumptions. In the active mode, the muscle is considered to be a source of force, attached with a flexible joint that generates no moments at the connecting points.

Joints and Articulating Facets

Biomechanical joints are very difficult to model exactly. The function of a joint is to allow specific movement and restrict other types of movement. Biomechanical joints, however, usually are somewhat compliant to these other types of movement. These joints could be modeled with the complex mechanical joints, but the equations describing such a joint are complex and time consuming. This model assumes that the biomechanical joints can be modeled with a system of nonlinear spring-damper pairs, compliant in tension while stiff in compression, fashioned in a way as to allow easy movement of the desired type for that particular joint, and to restrict other types of motion. The compliance in tension allows for the fact that the bone joints are not connective. Forces may be transmitted only in compression.

Note that the formulation of this model is based on qualitative information, not quantitative data. The anatomical features of the spine determined the configuration of the model. Accurate values for spring constants and damping coefficients are not yet known because of a lack of enough experiments. However, once a realistic and generally accepted model for the spine is developed, the data required to accurately model the spine will become needed and thus become available as research can then be directed toward that goal. The geometrical information used in this model (location of muscle attachments, joints, masses, etc.) are more easily measured. Hong and Suh [1] (1975) tested this data in a static model of the cervical spine, and the data used appears to be valid from the results obtained. Values for the spring constants and damping coefficients used in this model are based on estimates and should not be taken as accurate.

3. FORMULATION OF EQUILIBRIUM EQUATIONS

In the previous section, the anatomical features of the spine were modeled as a system of massless, nonlinear spring-damper pairs, rigid bodies, and applied forces. The single exception is the intervertebral discs, which were modeled using experimental data directly in an equation form obtained by curve fitting techniques. The discs can be viewed as a system of springs and dampers whose configuration is not known, but the overall effect of the system is known from the data. Therefore, dynamic analysis of the cervical spine now becomes the dynamic analysis of a system of nonlinear spring-damper pairs connected to a set of rigid bodies which may also be acted upon by applied forces.

The analysis of this elasto-dynamic system requires the formulation of the equilibrium equations for each rigid body. Since a rigid body in three-dimensional space has six degrees of freedom, six equations for each rigid body are required. These six equations are found by the simple relationships $F = ma$ and $T = I\alpha$. The forces and torques in the x, y, and z directions are summed and equated to the accelerations multiplied by the inertial components in those directions. Since acceleration is the second derivative of position, the six equations will be second order differential equations. Once these differential equations are formed, they must be solved at many points in time to establish the time relationship of the positions and velocities of the rigid bodies.

Formulation of Differential Equations Using Displacement and Differential Displacement Matrices

In order to calculate the forces and moments acting on the rigid bodies, the forces and positions of the spring-damper pairs, as well as the positions at which the applied forces act, must be determined. The positions of the geometric centers of each vertebra must also be known in order to find the forces and moments imposed by the intervertebral discs. It is possible to include the x, y, and z coordinates of each of these points as unknowns. However, there are only six equations for each rigid body necessary to determine its dynamic behavior. Additional constraint equations would have to be imposed to guarantee the rigid behavior of the rigid bodies. Obviously, if there are 8 rigid bodies and 100 spring-damper pairs, several hundred equations and unknowns would have to be solved simultaneously.

By utilizing the characteristics of the displacement matrix [2,3] and the differential displacement matrix [3,4] these several hundred equations may be reduced back to the original six equilibrium equations for each rigid body. A displacement matrix describes the displaced position of a rigid body in terms of its original position and a set of input quantities. Any point on that rigid body can be determined in the new position by multiplying the displacement matrix on the vector describing the original position of that point. The differential displacement matrix, or velocity matrix, similarly describes the velocity of a point in terms of the position of that point and another set of input quantities. The input quantities for the displacement matrix are the linear and angular displacements of the rigid body, and the inputs for the velocity matrix are the linear and angular velocities of the rigid body. Since these input quantities are the variables in the six second order equilibrium equations, no more constraining equations are needed [5].

In the model of the cervical spine, the intervertebral discs are modeled directly from data taken from actual spinal measurements. Since the geometric centers are the reference point for this data and the vertebrae are not symmetric in every plane, the center of mass may not coincide with the geometric center. Therefore, the displacement matrix is used to find the new position of the geometric center. The new positions of the geometric centers are inputs to a subroutine that calculates the forces and moments exerted on adjacent vertebrae due to that displacement, based on the data. These forces and moments are also summed with the forces and moments due to the spring-damper parts and applied forces in the equilibrium equations.

Each rigid body has six second order equilibrium equations. In the cervical spine model, there is a total of 48 equations to be solved simultaneously. The differential equations are solved on the computer by a routine called DASCRU [5]. DASCRU has proven to be a fast and accurate differential equation solving routine. However, it can solve only first order equations. The equilibrium equations must be transformed into first order equations in order to be solved in DASCRU. To illustrate how this is done, consider the following second order equation:

$$\ddot{A} = B\dot{A} + CA \quad .$$

Let the two unknowns (V_1 & V_2) in the solution be:

$$V_1 = A \quad , \text{ and}$$

$$V_2 = \dot{A} \quad .$$

Then the second order equation is transformed into two first order equations:

$$\dot{V}_2 = BV_2 + CV_1 \quad , \text{ and}$$

$$\dot{V}_1 = V_2 \quad .$$

In the cervical spine model, a set of 96 equations are produced in terms of 96 unknowns describing the position and velocity of each of the 8 rigid bodies. These 96 equations are then solved in DASCRU, starting at time t = 0 and progressing in small increments up to some final time. This is actually an integration process as the velocities are found by integration of the accelerations and the displacements are integrations of the velocities. The accuracy of these integrations increases as the time increment size decreases. The DASCRU routine automatically decreases the time increment size until sufficient accuracy is obtained (4 place accuracy).

The input to the system can be an initial velocity or position, or some forcing function. An initial velocity input would likely be used in the simulation of an auto accident. An initial position input might simulate a chiropractic adjustment. A forcing function input would be used to simulate muscular activity. Whatever the input, a useful form of output must be developed in order to fully utilize the information produced by the model.

4. FORM OF OUTPUT AND COMPUTER GRAPHICS

A great deal of information is produced during a dynamic simulation of the cervical spine, namely the linear and angular displacements and velocities of eight rigid bodies at many points in time. The accelerations of the rigid bodies are also calculated and can be made available. The model might calculate well over a million pieces of data in only a few seconds of simulation time. Obviously, the interpretation of these numbers into useful information could be a tedious and expensive task. Therefore, the ultimate value of a cervical spine model depends not only on the accuracy of the model, but also on the form of the information produced.

The particular application of the cervical spine model will determine what form of output is the most useful. The best form could indeed be the numerical results. For example, a researcher studying injury to the brain might know what values of acceleration would cause brain damage. For him, the most valuable information would be the maximum values of velocity and acceleration of the skull. These may be all that he is interested in. On the other hand, an engineer designing the interior of an automobile may only be interested in the path traveled by the skull during a simulated accident. For him, a graphic display of this motion may be sufficient. He probably has worked with accident simulation using mechanical dummies and high speed cameras, and could best work with an output that simulates a high speed film recording. Other types of research may require other forms of output from the cervical spine simulation either to relate to their current knowledge, or to provide new methods of analysis not possible with conventional techniques.

Sketching Method

In this method, a line sketch is drawn to represent each bone in the cervical spine. Then enough points on those lines are measured from the sketch so that when the measured points are connected with straight lines, an approximate of the original sketch is reproduced. Examples of the graphics produced by the sketching method are seen on the following pages.

The sketching method eliminates the biggest problem of the grid and X-ray methods — the need for large amounts of data to be collected and stored in order to define the bones. For example, the sketching method uses 233 points to sketch the skull, and much less for each vertebra. Although the sketching method simplifies the collection and storage of data, the hidden line elimination process cannot be used as the sketching lines do not define the boundaries of the body. If depth is required to make the 3-D effect clear, a varying intensity method is the most efficient method. In other words, the intensity of a line varies according to the distance from the observer.

While the sketching method is the simplest method of graphically displaying the cervical spine, it is also the most readable for most people. Motion pictures can also be generated using the same technique described in the X-ray method. As the dynamic simulation is being computed, the solution for each point in time can be drawn by the computer, either electronically or mechanically, and recorded with a movie camera or video tape. Rather than record every calculated solution, however, each second of simulation time is divided into a specific number of segments. If there were 200 segments per second, then the solutions for each multiple of .005 seconds would be drawn and recorded. The specific number of segments for each second depends on the type of playback device available and whether a real time viewing of the motion simulation is required. If so, the playback equipment must be capable of matching the recording intervals (200 frames per second in this case).

Example of Graphic Output

Consider the case of a passenger in an automobile. The car is struck on the left rear corner while stopped at an intersection. The passenger is wearing both lap and shoulder belts and the car is not equipped with head restraints. This "whiplash" can be simulated using this cervical spine model. Assume that the first thoracic vertebra is given a velocity at time $t = 0^+$. The velocities of the components of the cervical spine are of course still zero. [The head will snap back and to the left.] The muscles can be considered to be passive since the passenger will have no time to react to the accident.

Figure 4 is a sample of the graphical output of this simulation at various points in time. The solution has many points between these five pictures that would constitute a motion picture. The final motion picture records the total simulation in a form convenient for study later in time and not requiring the computer simulation to be repeated.

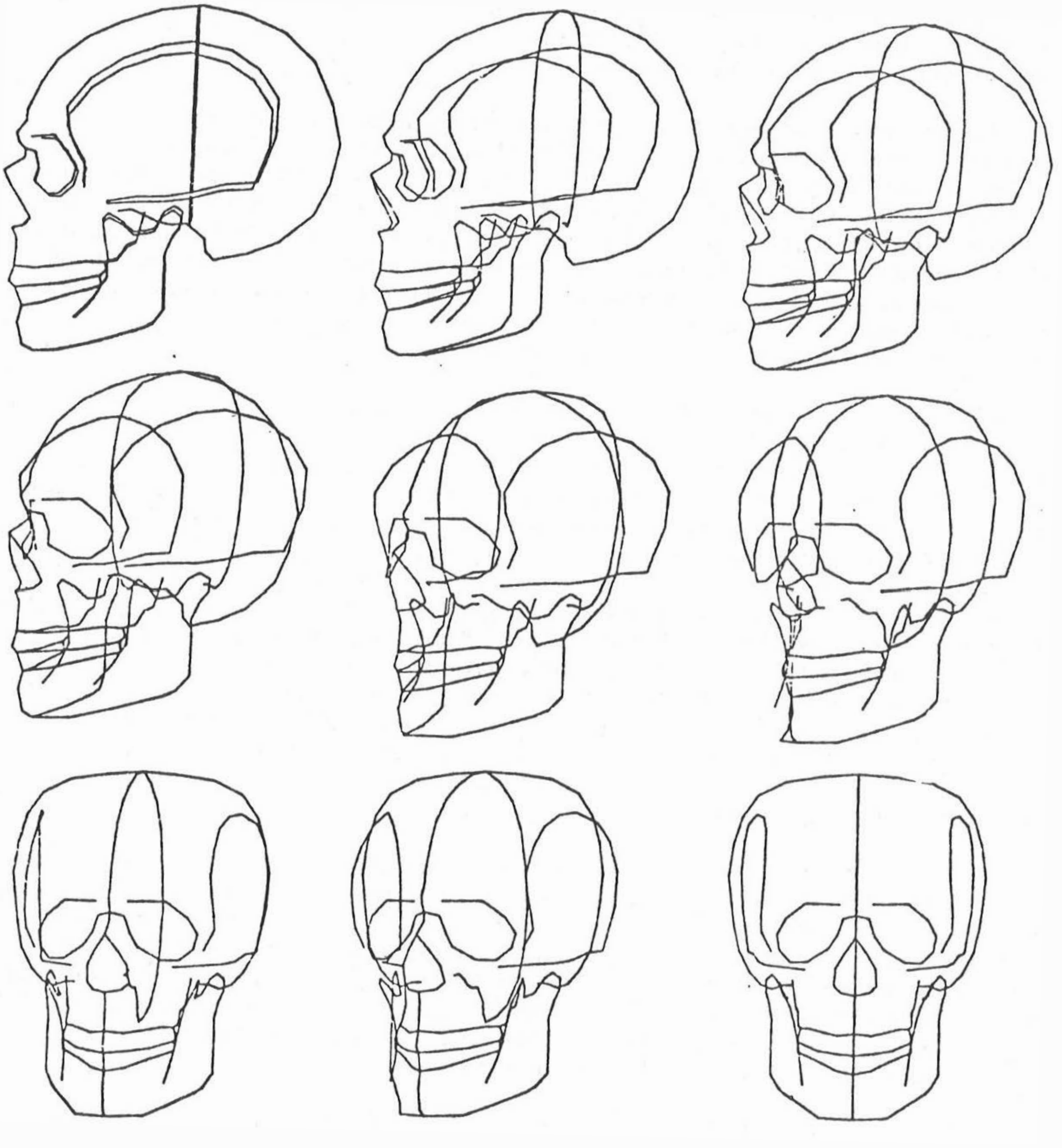
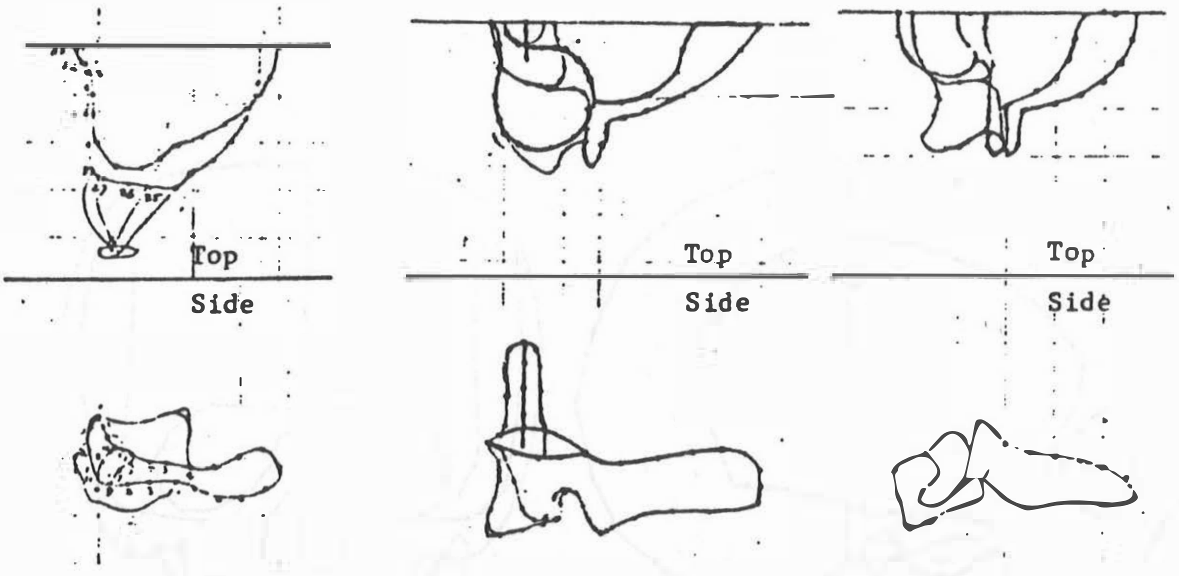
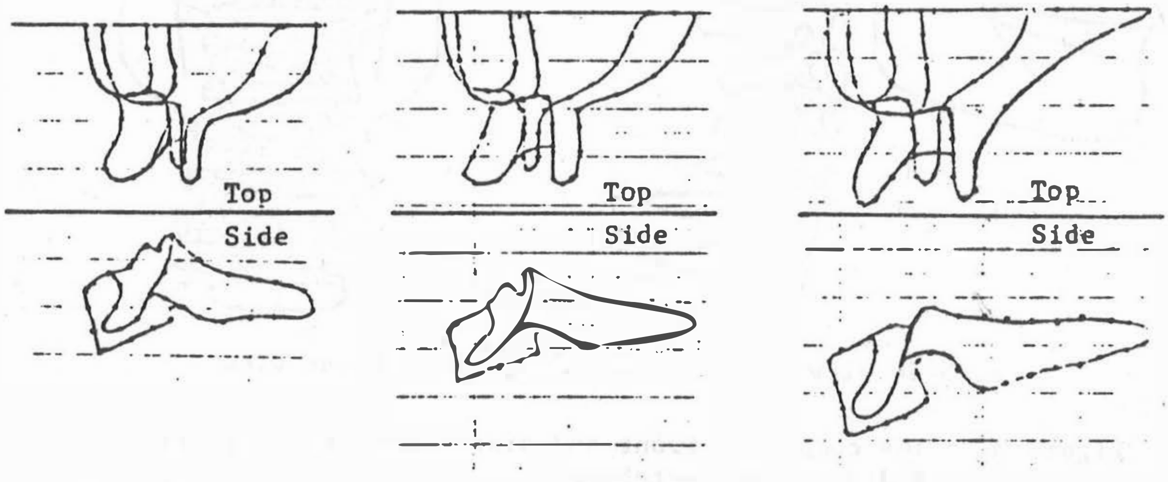


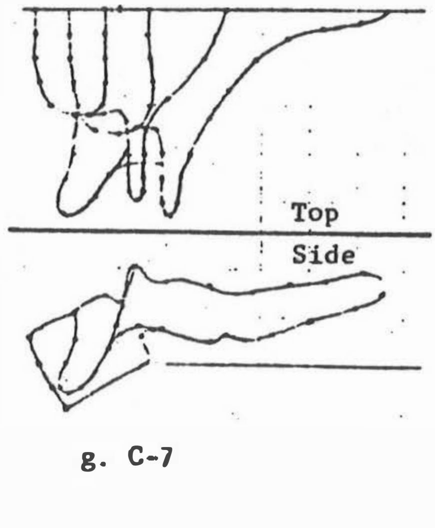
Figure 1. The different perspective views of skull.



a. Atlas b. Axis c. C-3

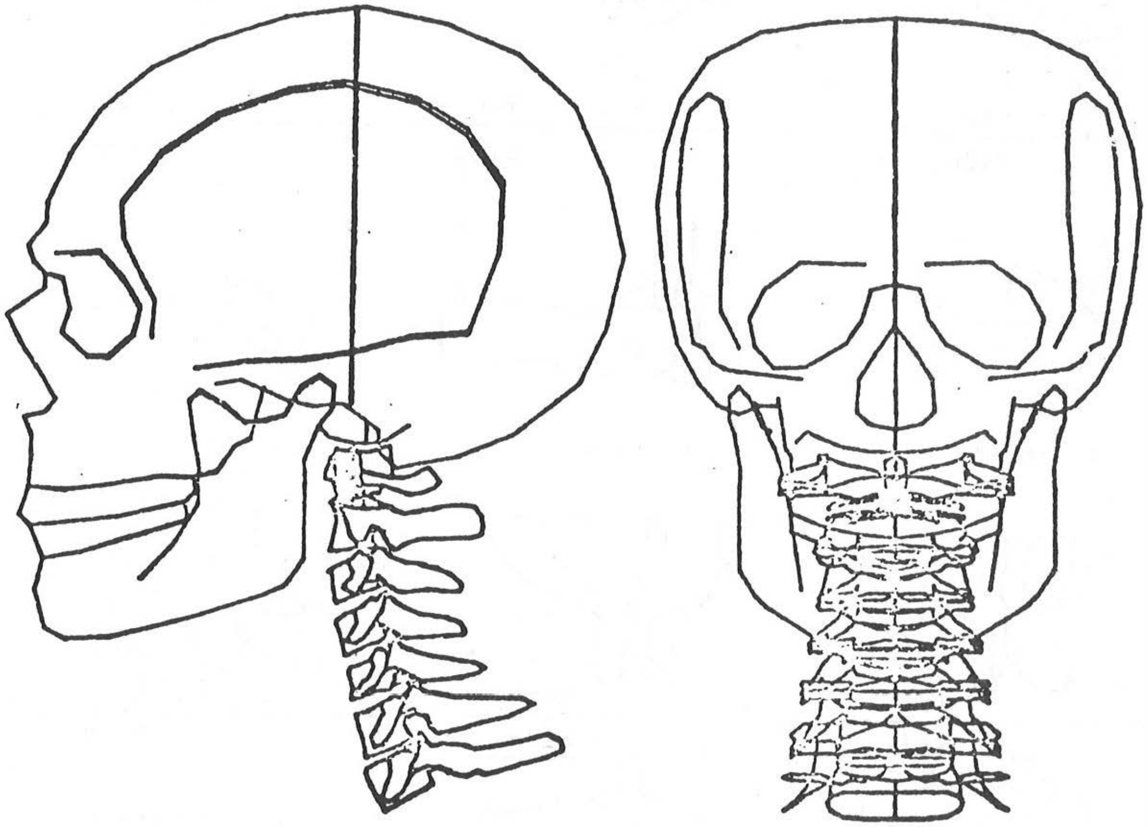


d. C-4 e. C-5 f. C-6



g. C-7

Figure 2. The sketches for the seven cervical vertebrae.



a. Side view

b. Front view

Figure 3. The completed front and side views of the skull and cervical vertebrae.

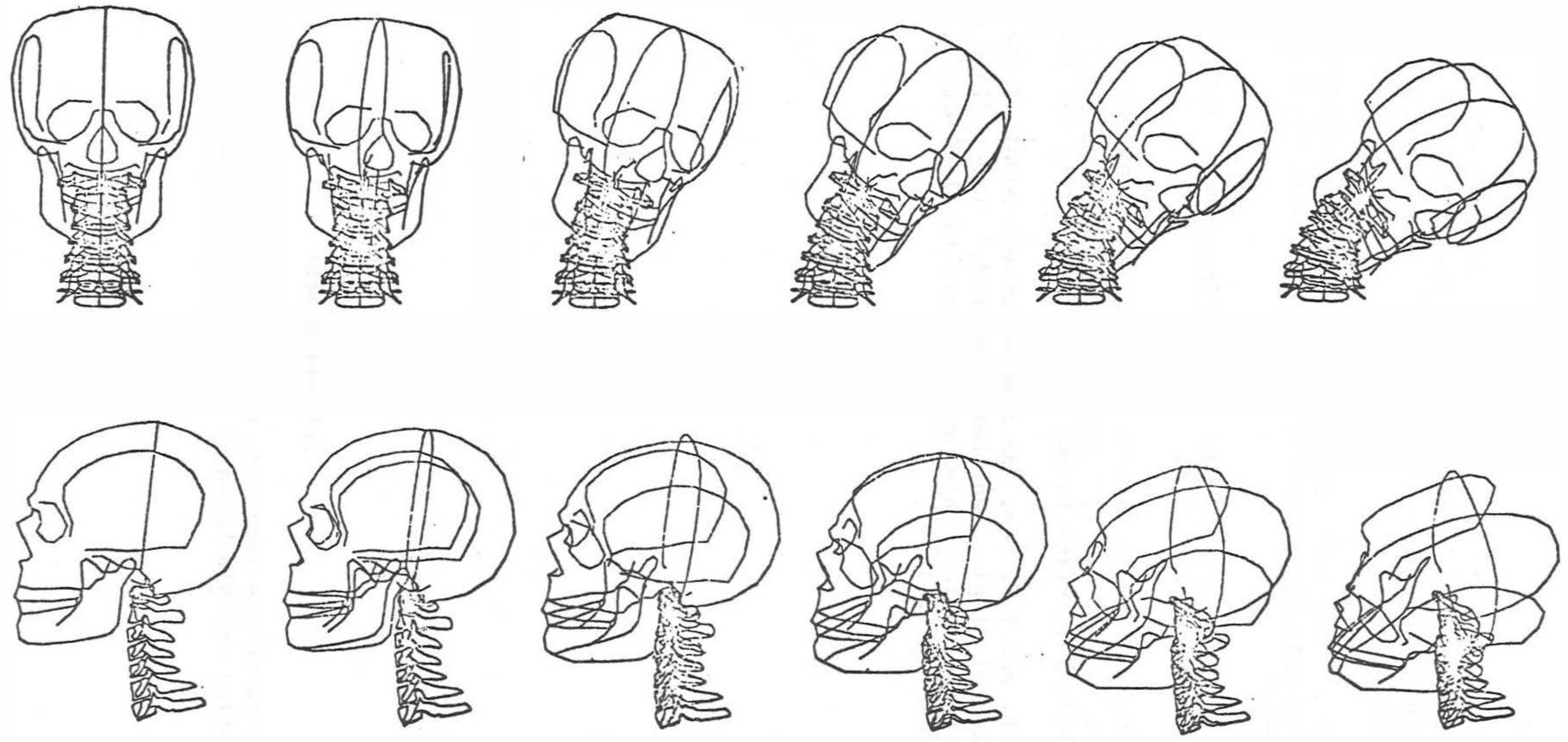


Figure 4. Simulated Motion of "Whiplash" Reaction.

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