

MATHEMATICAL RESEARCH TO OPTIMIZE THE SIDE-STRUCTURE  
OF PASSENGER CAR

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ABSTRACT

This presentation depicts a two-dimensional mathematical model for simulation of perpendicular side collisions. By means of this model the influence of structural rigidities on car-car collisions and on car-obstacle collisions is investigated. Trajectories of the occupants, accelerations and load values of the occupants are graphically represented as functions of front and lateral rigidity.

INTRODUCTION

Lateral collisions make up approximately 30% of the total number of traffic accidents, 70% of which consist of car-car collisions and 30% of which are car-rigid obstacle collisions [1].\*

The occupants in the car struck laterally are thereby considerably more endangered than the occupants in the striking car. For example, according to [1], lateral collisions result in injury severities  $\geq$  AIS3 twice as great as do frontal collisions.

The lateral collision in the compartment area is the most dangerous by reason that the forward structure of modern cars is substantially harder than the lateral structure; the greatest component of the impulse energy, therefore, must be absorbed by the lateral structure. The available deformation path of the forward structure is, therefore, barely utilized. According to [2], the lateral rigidity without the reinforcing influence of the door sill, which is rarely utilized for energy absorption in car-car collisions, amounts to only approximately one third to one half of the forward structure rigidity. The rigidity of the total structure sill-door-roof is about equal to that of the forward area.

The following contribution will examine, by means of a mathematical model for the simulation of perpendicular lateral impact, how changes in the structure rigidities influence the load values of the occupants upon car-car collisions and car-rigid obstacle collisions.

\* Numbers in parenthesis designate References at end of paper

## THE MATHEMATICAL MODEL

Fig. 1 shows the substituted mechanical system used for the following investigations. It is a two-dimensional seven-mass model with a total of 10 degrees of freedom. Only the calculation of the perpendicular lateral impact is possible.

The car-car collision, the barrier impact ( $C_1 \rightarrow \infty$ ) and the impact on a rigid obstacle ( $C_1, m_1 \rightarrow \infty$ ) can thereby be simulated.

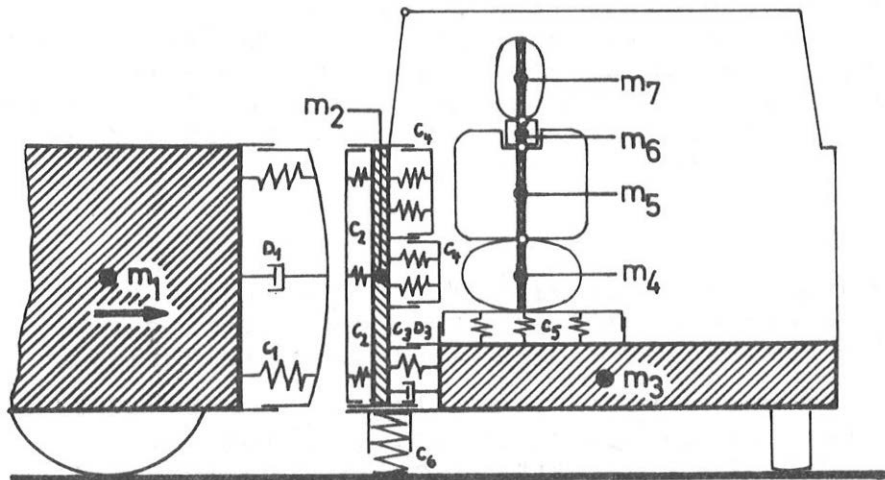


Fig. 1 mathematical model

$m_1$	mass of the striking car - 1 degree of freedom
$m_2$	mass of the lateral structure of the struck car - 1 degree of freedom (intrusion)
$m_3$	mass of the struck car - 2 degrees of freedom
$m_4 - m_7$	model of car occupant, 4 mass quantities (pelvis, trunk, neck, head), 3 swivel joints - 6 degrees of freedom
$C_1, D_1$	characteristics of the forward structure of the striking car
$C_2$	door rigidity
$C_3, D_3$	characteristics of the lateral structure of the struck car
$C_4, C_5$	characteristics of side and seat padding
$C_6$	tire characteristic at the left contact point

## CAR-CAR COLLISION

With the cars currently found in traffic, the ratio of lateral

structure rigidity to forward structure rigidity is approximately 1 to 2, i.e., the intrusion in the side is usually substantially greater than the deformation of the forward structure.

In the following, upon assumption of linear path of force characteristics in the car structures, investigation is made how a change of the rigidity ratio  $C_3/C_1$  affects the load values of the occupants of both cars.

A criterion for the load on an occupant in the striking car is thereby the acceleration of  $m_1$ . This is valid with assumption of the use of an optimal securing system (3-point automatic safety belt) at initial approach. This correlation, of course, exists only so long as the principle of the structurally stable occupant cell as survival space is maintained.

The load on the occupant in the struck car is determined by the Severity-Index (SI) and the Head-Injury-Criterion (HIC).

Figs. 2a and 2b show in three-dimensional presentation the dependence of the load on the occupant of the struck car (SI(chest)), and of the load on the occupant of the striking car ( $a(m_1)$ ) on forward and lateral rigidity.

The collision speed for all curves is 13.89m/sec and the mass ratio is 1. The actual condition shown in the diagrams is

$$C_3(\text{standard})/C_1(\text{standard}) = 1/2$$

Figs. 2a,b show that a reduction of the forward structure rigidity  $C_1$  has basically positive results for the load values on occupants of both cars.

With maintenance of the principle of the inherently stable occupant cell, a reduction of  $C_1$  is possible only by a corresponding increase of the deformation zone of the forward structure.

The reduction of the occupant load in the struck car is substantially greater than the reduction of the occupant load in the striking car. The reason is the unfavourable situation of the actual condition and the dependence of the impact speed of the occupant in the struck car on the structural rigidities (see below).

Since the principle of the structurally stable occupant cell for the struck car is not maintained for the actual condition of the lateral collision, a change in lateral rigidity differently affects the occupants of both cars.

The risk of injury increases for the occupant of the striking car. Doubling  $C_3$  with respect to the actual condition increases the maximum acceleration of the striking car and, thus, the load on an occupant, by approximately 25%.

In contrast, the load on an occupant in the struck car is reduced considerably with increased lateral rigidity. Doubling of  $C_3$  with respect to the actual condition reduces SI(chest) by 100%. If a certain "optimal" lateral rigidity is exceeded, however, the occupant load increases again.

The path of the curve may be explained as follows:

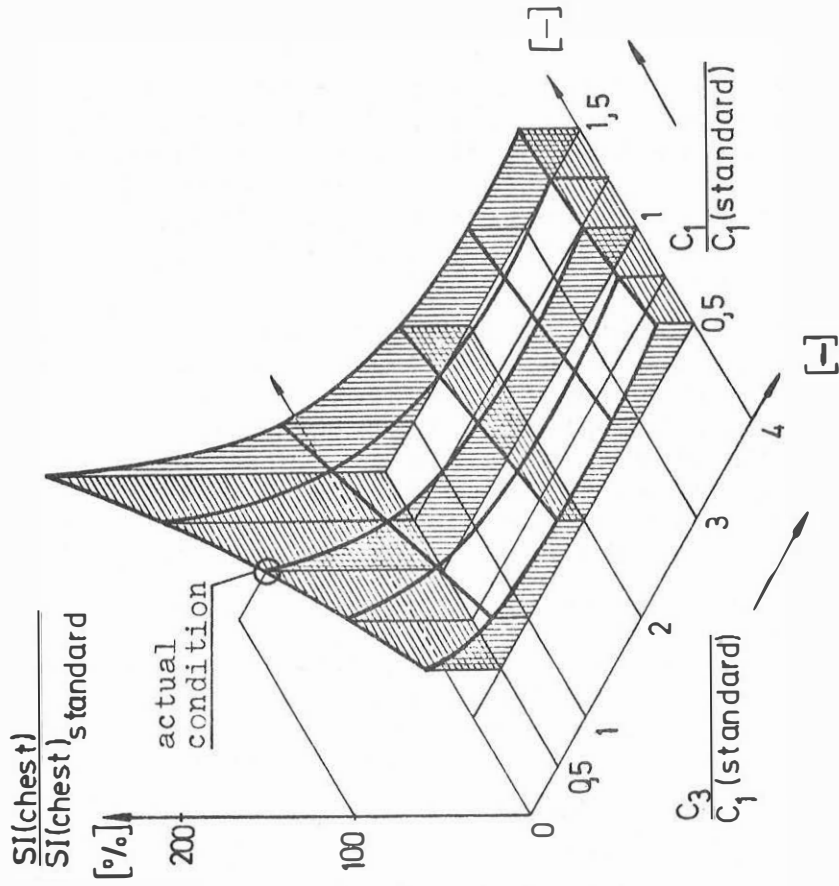


Fig. 2b relative severity-index versus relative side- and front-stiffness; struck-car-occupant

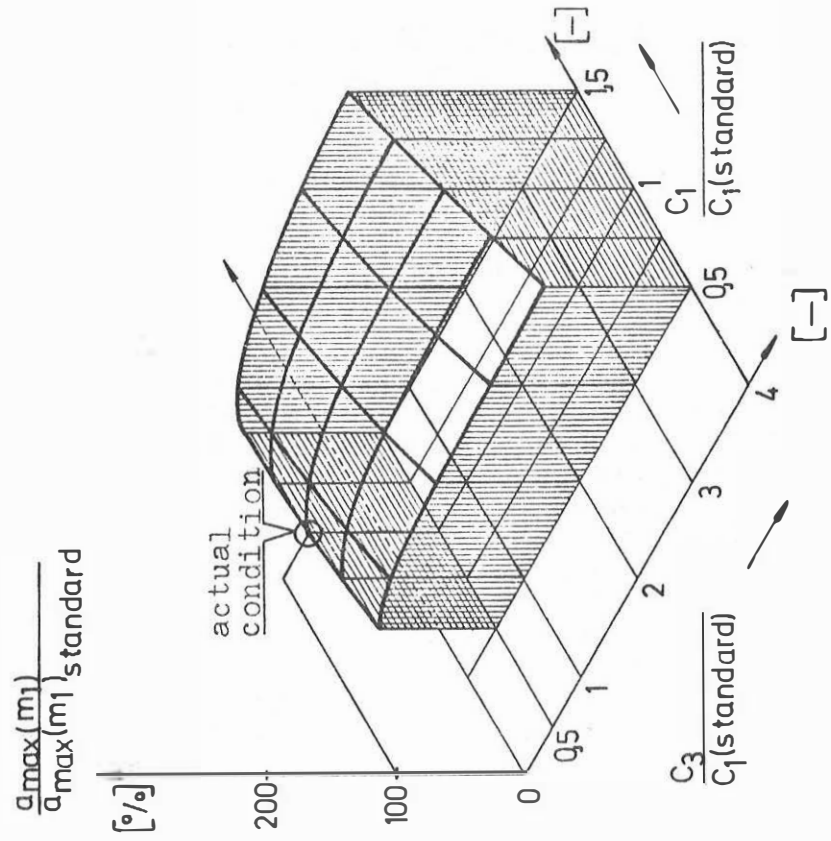


Fig. 2a relative deceleration versus relative side- and front-stiffness; striking-car

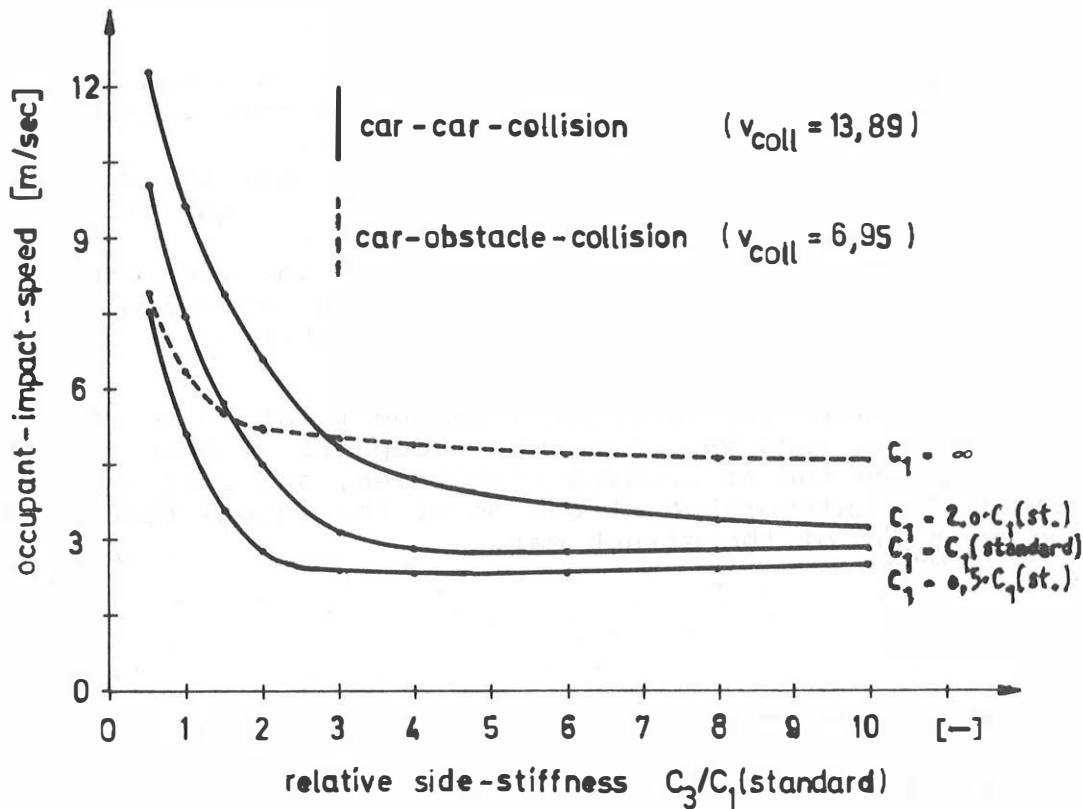


Fig. 3 occupant-impact-speed versus relative side-stiffness for car-car-collision and car-fixed obstacle-collision

An alteration of the lateral structure rigidity (and also of the forward rigidity) strongly affects, near actual conditions, the interior impact velocity of the occupant against the lateral cushioning. Fig. 3 shows the interior impact speed, which results from the addition of intrusion speed of the side and change of speed of the struck car, in dependence on  $C_3$ . In the case of a soft lateral structure, the occupant is struck in his original sitting position with nearly collision speed by the intruding structure. This high interior impact speed naturally leads to high occupant load values. The impact speed decreases rapidly with increasing lateral rigidity and asymptotically approaches the speed of the struck car at the time of the interior impact. The load reduction on the occupant of the struck car is also directly based on the great reduction of the interior impact speed. The increase upon exceeding of a certain value of  $C_3$  is limited by the construction of the mathematical model: An increase of  $C_3$  has the same effect on the momentum exchange between occupant and side mass  $m_2$  as a direct increase of  $m_2$ .

As  $C_3 \rightarrow \infty$ ,  $m_2$  and  $m_3$  approach each other; the mass ratio at interior impact is, therefore, substantially more unfavorable for the occupant than at a small value of  $C_3$ .

Since reinforcement of the side is possible only through an increase of the side mass, this negative effect may well be transferred into actual effect.

The indirect mass increase of  $m_2$  results in the increase of load values on an occupant in the struck car upon exceeding of the optimal lateral rigidity. Beforehand, the substantial reduction of the interior impact speed prevails.

Table 1 again shows the percentual change relative to the actual condition of the load values on the occupants of both vehicles, in dependence on the structural rigidities, and once again emphasizes the unfavorable situation of the actual condition for the occupant of the struck car.

$\frac{C_1}{C_1(\text{standard})}$	striking-car	struck-car	
	$\frac{a(m_1)}{a(m_1)_{\text{standard}}}$	$\frac{SI(\text{chest})}{SI(\text{chest})_{\text{standard}}}$	$\frac{HIC}{HIC_{\text{standard}}}$
0,5	89,2 %	41,8 %	39,0 %
0,75	96,1 %	69,0 %	64,4 %
1,0	100,0 %	100,0 %	100,0 %
1,25	102,4 %	140,2 %	176,0 %
1,5	104,1 %	190,1 %	291,1 %
$\frac{C_3}{C_1(\text{standard})}$			
0,5	100,0 %	100,0 %	100,0 %
0,75	114,0 %	64,7 %	62,3 %
1,0	123,7 %	47,8 %	43,8 %
2,0	142,3 %	26,5 %	23,3 %
8,0	161,5 %	39,0 %	41,8 %
$\infty$	165,3 %	69,1 %	65,8 %

actual condition

Table 1 relative load values on the occupants of both vehicles, in dependence on the structural rigidities

Fig. 4 shows the occupant's motion sequence with soft lateral structure -4a- and with hard lateral structure -4b-. The coordinate system lies here in the right tire contact point of the struck car. The different motion sequences clearly demonstrate the influence of the reinforcement of the side on the occupant's motion sequence and, furthermore, point out the following problem:

With a hard side, the danger of a head impact onto the side window is considerably greater than with a soft side, with which the occupant is pushed away from the side window by the intruding structure.

Fig. 6a shows the acceleration processes of the occupant's bodily parts belonging to the motion sequences. Since the distance of the occupant to the side padding is reduced more slowly with a hard side structure than with a soft structure, the acceleration increase occurs transposed in time. This effect has already been experimentally demonstrated in [3] as well. With a hard side, the acceleration peaks decrease at impact on the interior padding; however, because of the increased danger of head impact onto the side window, there is an increased risk of head injuries.

At head impact, considerable angular acceleration occurs, and the acceleration peaks quickly exceed the permissible maximum values. Since, upon side collisions, the head is the most endangered part of the body in any case, this effect must be considered when optimizing the lateral structure.

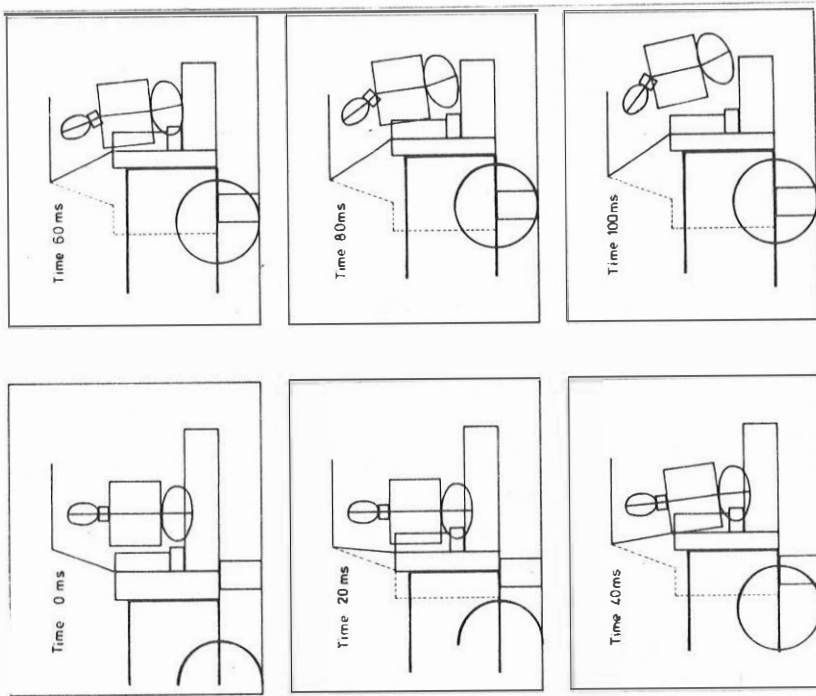
#### CAR-FIXED OBSTACLE-COLLISION

Upon impact onto a rigid obstacle, the entire kinetic energy of the collision must be absorbed by the lateral structure, in contrast to a car-car collision. Although the intrusion is reduced by reinforcement, the maximum car deceleration is increased inversely proportionally to the intrusion (with linear characteristic).

$$\frac{a_{\max}(m_3)}{a^*_{\max}(m_3)} = \frac{\text{Intrusion}^*}{\text{Intrusion}} = \sqrt{\frac{C_3}{C_3^*}}$$

Fig. 7 shows for the side collision with a rigid obstacle the dependence of the occupant load values on the lateral rigidity  $C_3$ . The marked decrease of the load values with increasing lateral rigidity upon car-car collisions is not observed here. The values remain nearly constant up to a certain value of lateral rigidity, and then increase strongly and progressively thereafter.

The difference results from the fact that the interior impact speed is reduced only slightly (Fig. 3, dotted line) upon an



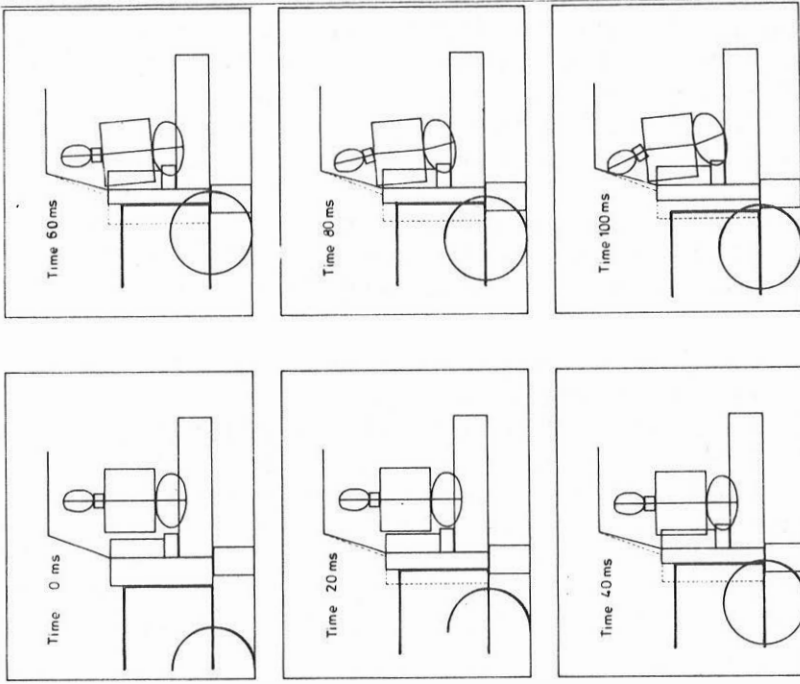
**Fig. 4a**

car-car-collision

occupant motion sequence

soft side-structure

$C_3 = C_3(\text{standard}) = 100\%$   
 $SI(\text{chest})/SI(\text{chest})\text{standard} = 100\%$   
 $HIC/HIC\text{standard} = 100\%$   
 $\ddot{\alpha}(\text{head})/\ddot{\alpha}(\text{head})\text{standard} = 100\%$



**Fig. 4b**

car-car-collision

occupant motion sequence

hard side-structure

$C_3 = 16 \cdot C_3(\text{standard})$   
 $\uparrow 41\%$   
 $\uparrow 42\%$   
 $\uparrow -110\%$



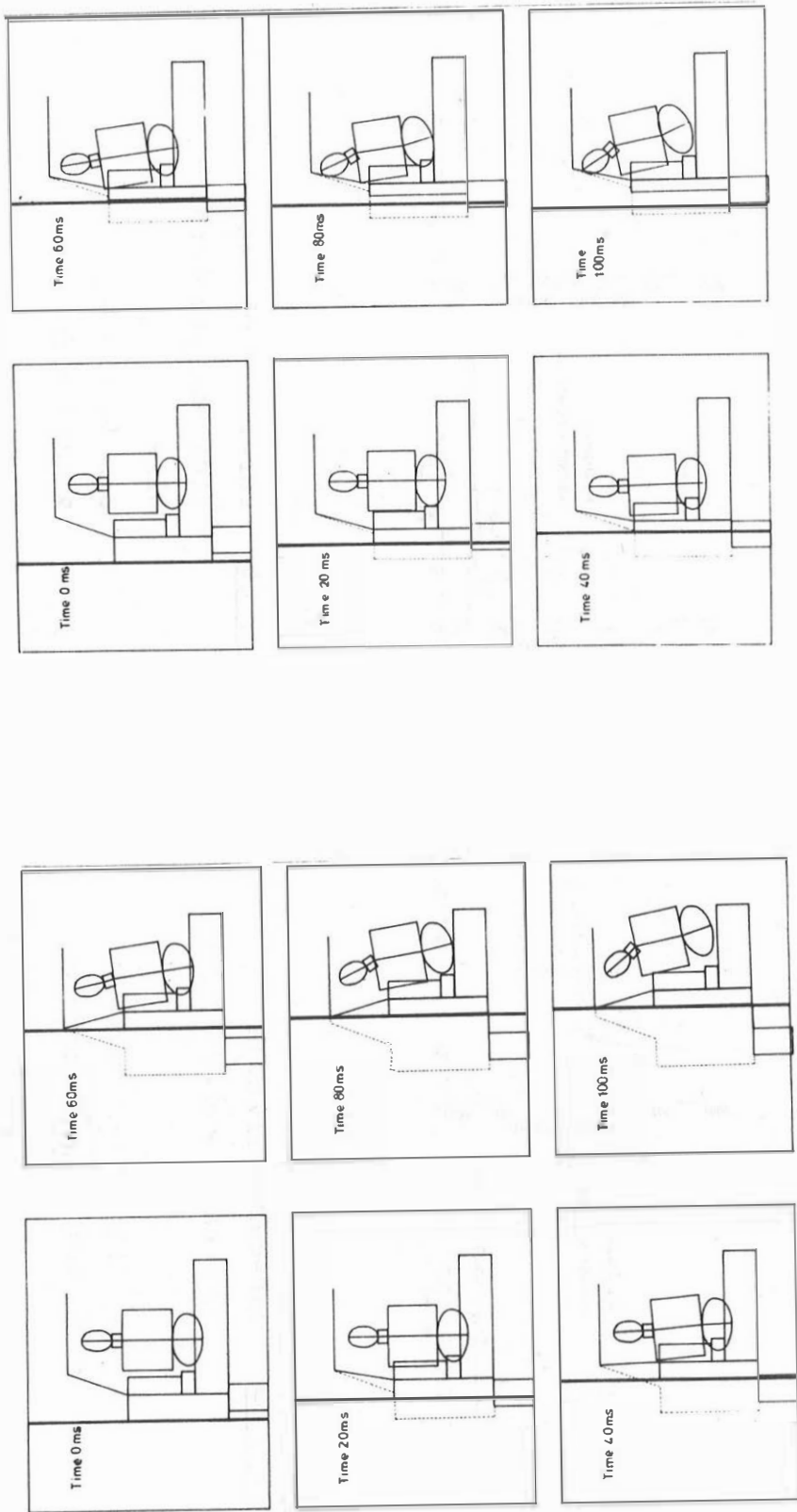


Fig. 5a car-fixed obstacle-collision

occupant motion sequence

soft side-structure

$C_3 = C_3(\text{standard}) = 100\%$   
 $SI(\text{chest})/SI(\text{chest})_{\text{standard}} = 100\%$   
 $HIC/HIC_{\text{standard}} = 100\%$   
 $\ddot{\alpha}(\text{head})/\ddot{\alpha}(\text{head})_{\text{standard}} = 100\%$

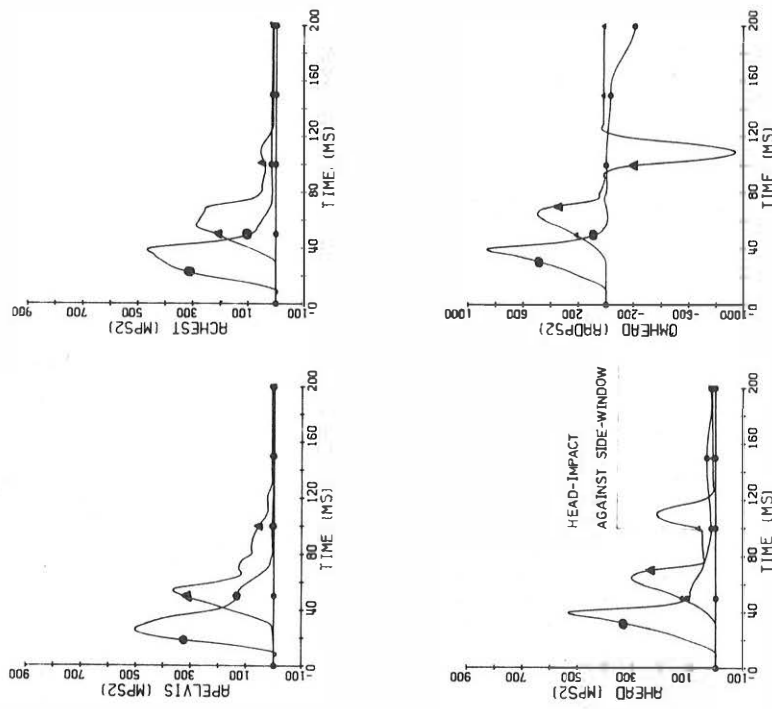
Fig. 5b

car-fixed obstacle-collision

occupant motion sequence

hard side-structure

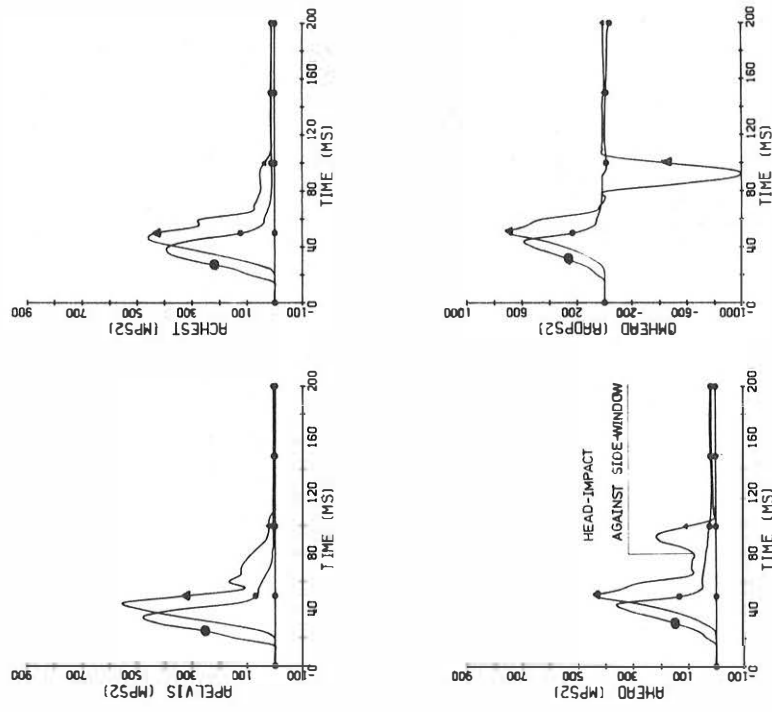
$C_3 = 16 \cdot C_3(\text{standard})$   
 $\uparrow 145\%$   
 $\uparrow 163\%$   
 $\uparrow -164\%$



**Fig. 6a** car-car-collision

occupant-accelerations versus  
time

- soft side-structure
- ▼ hard side-structure



**Fig. 6b** car-fixed obstacle-collision

occupant-accelerations versus  
time

- soft side-structure
- ▼ hard side-structure

increasing lateral rigidity. The negative influence of the indirect mass increase is therefore predominant from the beginning. Furthermore, it occurs more markedly than with a car-car collision, since the side mass  $m_2$  supports itself via the door rigidity  $C_2$  against an infinitely large mass with a rigid characteristic.

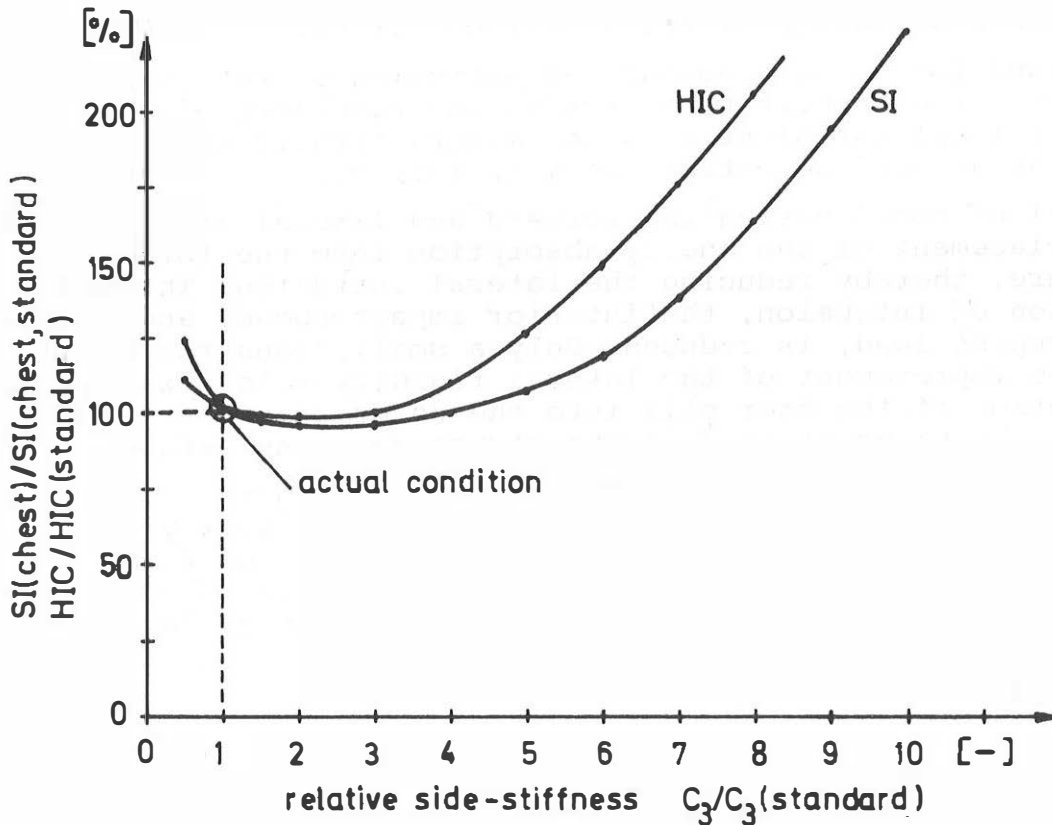


Fig. 7 car-fixed obstacle-collision  
relative Severity-Index and relative Head-Injury-Criterion versus relative side-stiffness

Fig. 5 shows the occupant's motion sequence upon impact against a rigid obstacle, with a soft and with a hard lateral structure; and Fig. 6b shows the respective acceleration process of the occupant's bodily parts.

One recognizes that, in contrast to car-car collisions, the peak values of the occupant's accelerations upon interior impact are increased, and that the danger of head impact onto the side

window increases for the same reasons as with car-car collisions with increasing lateral rigidity.

#### SUMMARY

The calculations for a perpendicular side collision with linear structural characteristics has shown that changes in structure rigidities affect the occupant of the struck car much more markedly than the occupant of the striking car because of the unfavorable situation of the actual condition.

The demand for an enlargement and softening of the forward structure with a positive effect on the occupants of both cars is trivial and unrealistic, since modern structures already represent almost the optimum in this respect.

The goal of coordinating the forward and lateral structures must be displacement of the energy absorption into the forward structure, thereby reducing the lateral intrusion. Through reduction of intrusion, the interior impact speed, and, therefore, the occupant load, is reduced. Only a small, constructively possible improvement of the lateral rigidity - for example by integration of the door sill into the deformation structure - reduces the occupant load in the struck car considerably.

The load on the occupant in the striking car increases only insignificantly with utilization of an optimal safety belt system. A worsening of the conditions at lateral impact on a rigid obstacle is thereby not to be expected, since an increase of the occupant load values occurs only at extremely high lateral rigidity.

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